# *Declarative Machine Learning and Data Mining*

Luc De Raedt

*CP for Analytics Workshop, Cork, 2015*

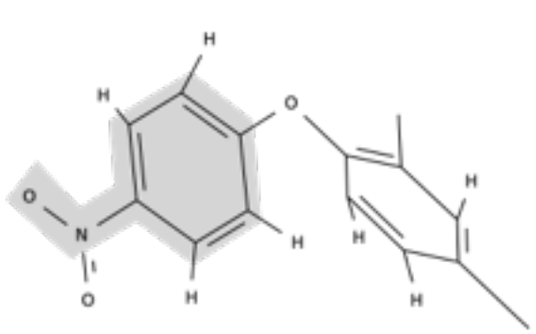
*Joint work with Angelika Kimmig, Siegfried Nijssen, Tias Guns and many others*

# Motivation

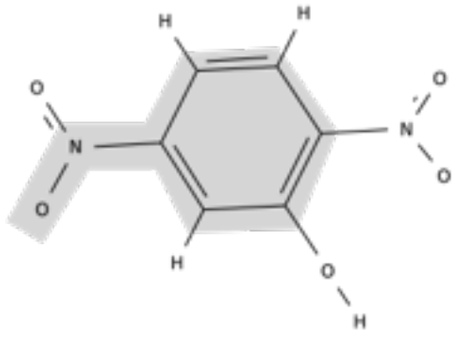
- Use of declarative methods, a new trend in machine learning and data mining
- Two sources of inspiration
  - Stephen Boyd on Convex Optimisation for Statistical Learning
  - Helmut Simonis in CP
- Introduce two types of declarative languages for ML & DM, logic / symbolic oriented rather than pure linear algebra
  - CP4IM & MiningZinc — CP for pattern mining
  - Probabilistic Programming — in ProbLog

# Declarative Mining

# Patterns



Mutagenic



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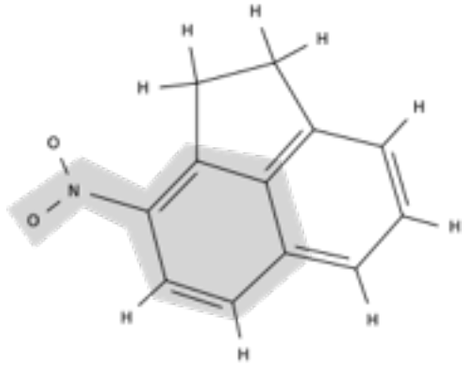

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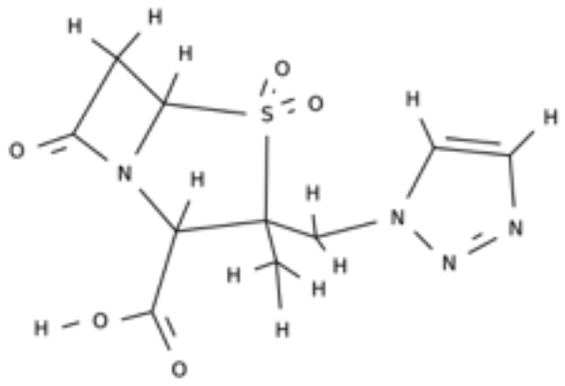
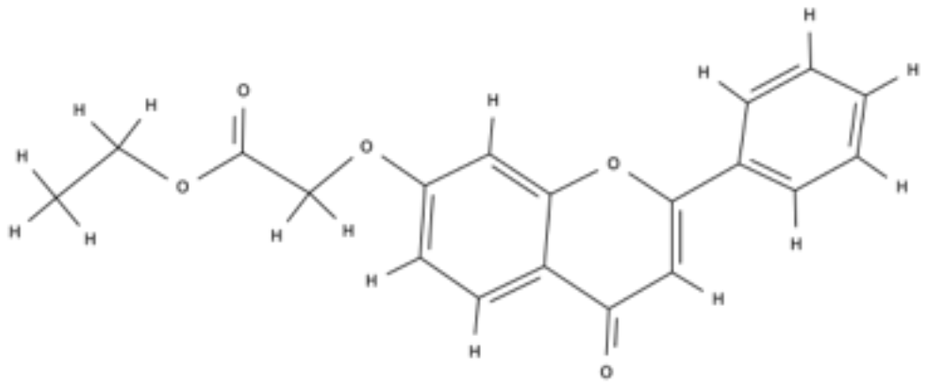


Clean



Clean

# Pattern Mining

## A. frequent pattern

- which patterns are frequent ?

$$Th(\mathcal{L}, Q, \mathcal{D}) = \{p \in \mathcal{L} | Q(p, \mathcal{D}) = true\}$$

## B. Correlated pattern mining = subgroup discovery

- which patterns are significant w.r.t. classes ? all patterns ? k-best patterns ?

$$Th(\mathcal{L}, Q, \mathcal{D}) = \arg_{p \in \mathcal{L}} \max_k \phi(p, \mathcal{D})$$

## C. pattern set mining

- which pattern *set* is the *best* concept-description for the actives ? for the inactives ?

$$Th(\mathcal{L}, \mathcal{Q}, \mathcal{D}) = \{P \subseteq \mathcal{L} | \mathcal{Q}(P, \mathcal{D}) = true\}$$

# Itemset mining

Data

Frequent patterns

4

2

3

# A. Frequent Itemset Mining

Given

- $\mathcal{I} = \{1, \cdots, NrI\}$
  set of items
- $\mathcal{T} = \{1, \cdots, NrT\}$
  set of transactions identifiers
- $\mathcal{D} = \{(t, I) | t \in \mathcal{T}, I \subseteq \mathcal{I}\}$
  Dataset
- $Items \subseteq \mathcal{I}$ and $Trans \subseteq \mathcal{T}$

Find $Items$ such that
$|covers(Items, \mathcal{D})| > freq$

where $covers(Items, \mathcal{D}) =$
$\{t \in \mathcal{T} | (t, I) \in \mathcal{D}$ and $Items \subseteq I\}$

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$\{t \in \mathcal{T} | (t, I) \in \mathcal{D}$ and $Items \subseteq I\}$

```
int: Freq;
int: NrI;
int: NrT;

array[1..NrT] of set of 1..NrI: D;

var set of 1..NrI: Items;
var set of 1..NrT: Trans;

constraint card(Trans) > Freq;
constraint Trans = covers(Items, D);

solve satisfy;


function var set of int: cover(Items, D) =
let {
        var set of int: Trans,
        constraint forall (t in ub(Trans))
        (t in Trans ↔ Items subset D[t])
} in Trans;
```

# Frequent Itemset Mining

math like notation

user defined functions and constraints

solver independent (standardized)

efficiently solvable

```
int: Freq;
int: NrI;
int: NrT;

array[1..NrT] of set of 1..NrI: D;

var set of 1..NrI: Items;
var set of 1..NrT: Trans;

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let {
        var set of int: Trans,
        constraint forall (t in ub(Trans))
        (t in Trans ↔ Items subset D[t])
} in Trans;
```
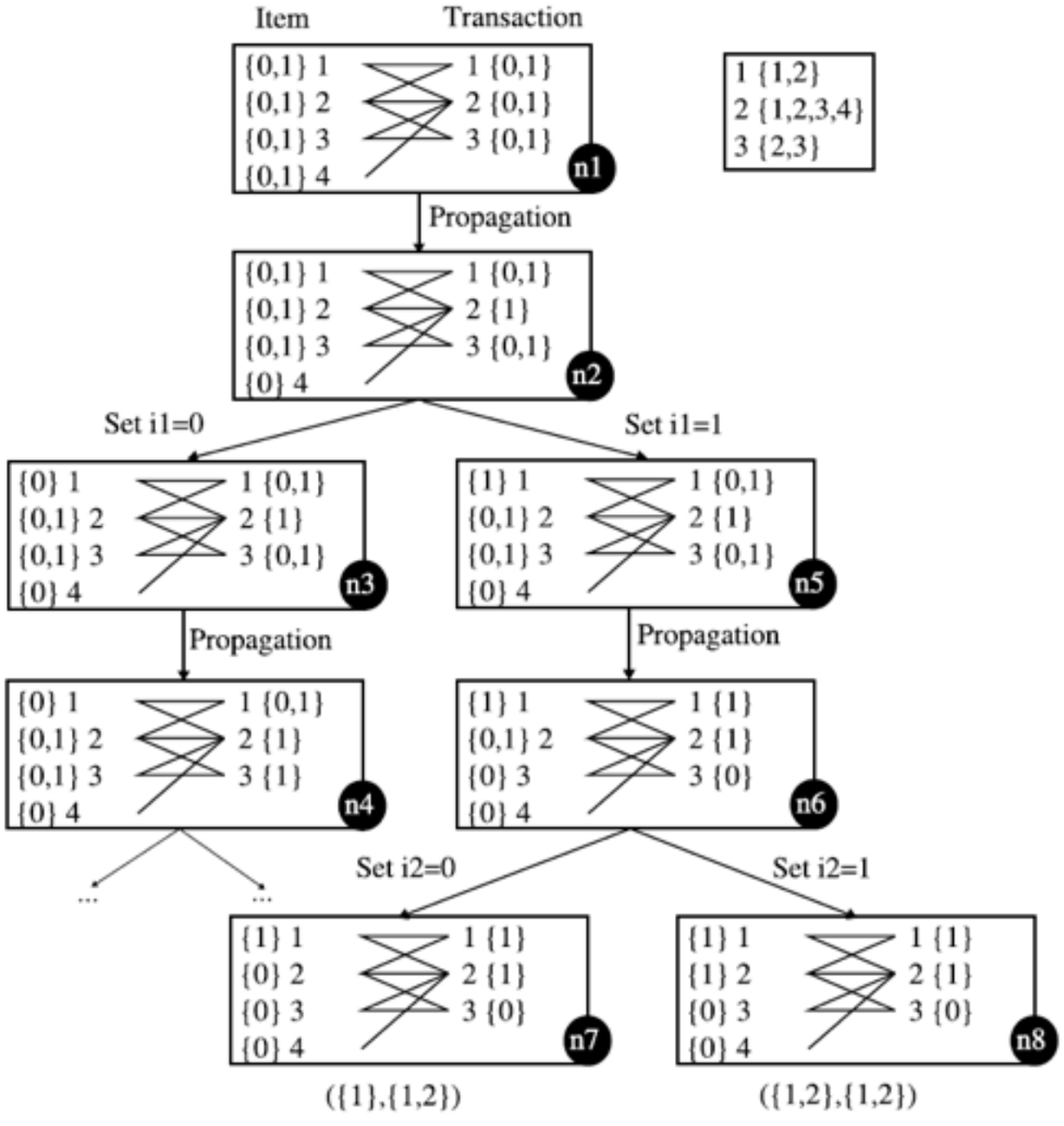

# Encoding in Zinc

```
int: Freq;
int: NrI; int: NrT;
array [1..NrT] of set of int: D;

array [1..NrI] of var bool: Items;
array [1..NrT] of var bool: Trans;

constraint % encode D: every Trans complement has no supported Items
  forall(t in 1..NrT) (
     Trans[t] <-> sum(i in 1..NrI) ( Items[i]*(1 - (i in D[t])) ) <= 0
  );

constraint % frequency: every Item is supported by sufficently many Trans
  forall(i in 1..NrI) (
     Items[i] -> sum(t in 1..NrT) ( Trans[t]*(i in D[t]) ) >= Freq
  );

solve satisfy;
```

$$\forall t : T_t = 1 \Leftrightarrow \sum_i I_i(1 - D_{ti}) = 0$$

$$\sum_t T_t \geq minsup \quad \textbf{iff} \quad \forall i : I_i = 1 \Rightarrow \sum_t T_t D_{ti} \geq minsup$$

# Resulting Search Strategy akin to Zaki's Eclat [KDD 97]



# Closed Itemset Mining

```
int: Freq;
int: NrI;
int: NrT;

array[1..NrT] of set of 1..NrI: D;

var set of 1..NrI: Items;
var set of 1..NrT: Trans;
constraint card(Trans) > Freq;
constraint Trans = covers(Items, D);
constraint Items = cover_inv(Trans, D);
solve satisfy;
```

```
function var set of int: cover_inv(Trans,D)=
 let {
        var set of int: Items,
        constraint forall (i in ub(Items))
      (i in Items ↔ Trans subset D'[i] )
} in Items;
```

```
function var set of int: cover(Items, D) =
let {
        var set of int: Trans,
        constraint forall (t in ub(Trans))
      (t in Trans ↔ Items subset D[t] )
} in Trans;
```

# Further Constraints

* exact coverage :
  t in Trans <-> Items subset D[t]

* freq:
  i in Items -> card(Trans intersect D'[i]) >= Freq

* maximal:
  i in Items <-> card(Trans intersect D'[i]) >= Freq

* closed:
  i in Items <-> Trans subset D'[i]

* delta-closed:
  i in Items <-> card(Trans intersect D'[i]) <= Delta*card(Trans)

# Top-k Correlated Pattern Mining

- $\mathcal{D}$ now consists of two datasets, say $P$ and $N$
- a correlation function $\phi(p, \mathcal{D})$, e.g., $\chi^2$
- $Th(\mathcal{L}, Q, \mathcal{D}) = \arg_{p \in \mathcal{L}} \max_k \phi(p, \mathcal{D})$

# Modeling perspective

```
int: NrI; int: NrT; int: Freq;
array[1..NrT] of set of int: D;
set of int: pos; set of int: neg;

var set of 1..NrI: Items;
var set of 1..NrT: Trans;

constraint Trans = cover(Items, D);
constraint Items = cover_inv(Trans, D);

solve maximize
        card(Trans intersect pos) – card(Trans intersect neg)
```

accuracy

Alternative opt. functions, for example:

```
solve maximize chi2(Trans, pos, neg);
```

with:

```
function float: chi2(Trans, pos, neg)
```

# Correlation function

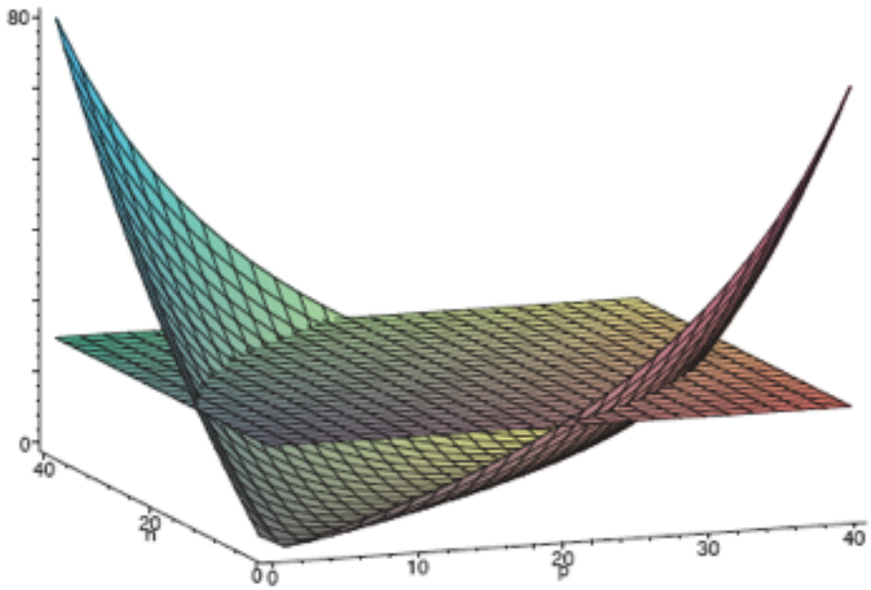
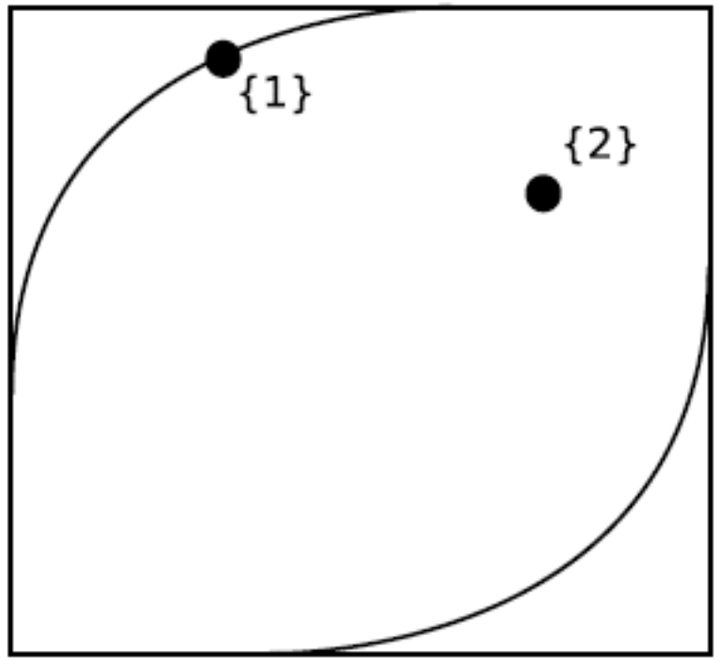



**Figure 1: A plot of the $\chi^2$ scoring function, and a threshold on $\chi^2$.**



# Generality

| | LCM [15] | MAFIA [6] | ExAMiner [4] | DualMiner [5] | DDPrune [12] | CP4IM |
|---|---|---|---|---|---|---|
| Constraints on data | | | | | | |
| Minimum frequency | X | X | X | X | | X |
| Maximum frequency | | | | X | | X |
| Emerging patterns | | | | | | X |
| Condensed Representations | | | | | | |
| Maximal | X | X | | X | | X |
| Closed | X | X | | | | X |
| $\delta-$Closed | | | | | | X |
| Constraints on syntax | | | | | | |
| Max/Min total cost | | | X | X | | X |
| Minimum average cost | | | X | | | X |
| Max/Min size | X | X | X | X | | X |
| Constraints on labelled data | | | | | | |
| Minimum correlation | | | | | X | X |
| Maximum correlation | | | | | | X |

[L. De Raedt, T. Guns, S. Nijssen, AAAI 2010]

# Declarative constraint-based mining

Language goals:

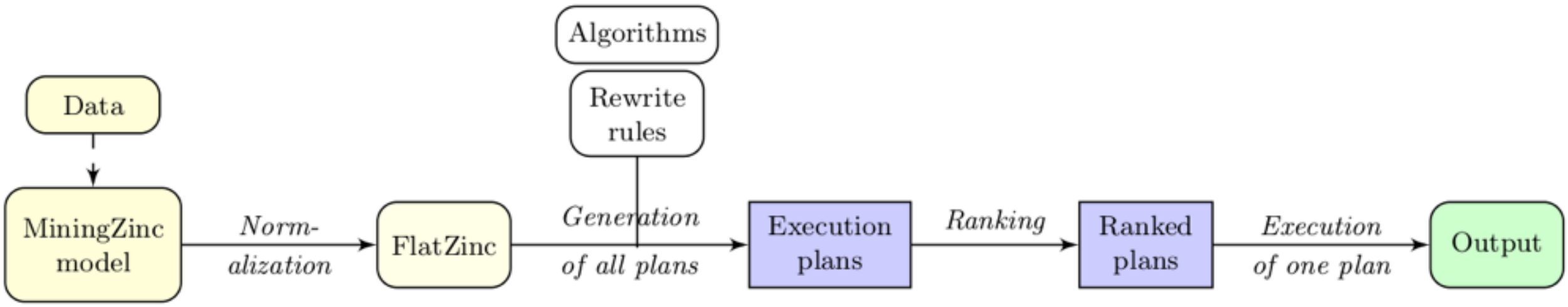
- high-level notation (similar to paper definitions)
- solver-independent
- user-defined abstractions


- mathematical-like language (Zinc)
- many solvers
- can define custom predicates *and functions*

=> MiningZinc

[T. Guns, A. Dries, S. Nijssen, G. Tack, L. De Raedt, IJCAI 2013]

# Toolchain




1) Normalize to FlatZinc *(do not flatten lib_itemsetmining.mzn yet)*
2) Apply rewrite rules to:
   - add redundant constraints
   - detect (partial) applicability of specialised algorithms
   - tailor to constraint solvers
3) Collect all feasible rewrite combinations = execution plans
4) Heuristically rank + execute a plan

```
var set of 1..NrI: Items; array[int] of set of int: TDB;
constraint card(cover(Items, TDB)) >= 20;
constraint card(cover(Items, TDB)) =< 40;
solve satisfy;
```

Three categories of execution plans:

A) Specialised algorithms only

- Eclat-maxfreq(TDB, 20, 50)
- LCMv2(TDB, 20) + maxcover(Items, TDB, 40)

B) Hybrid decomposition

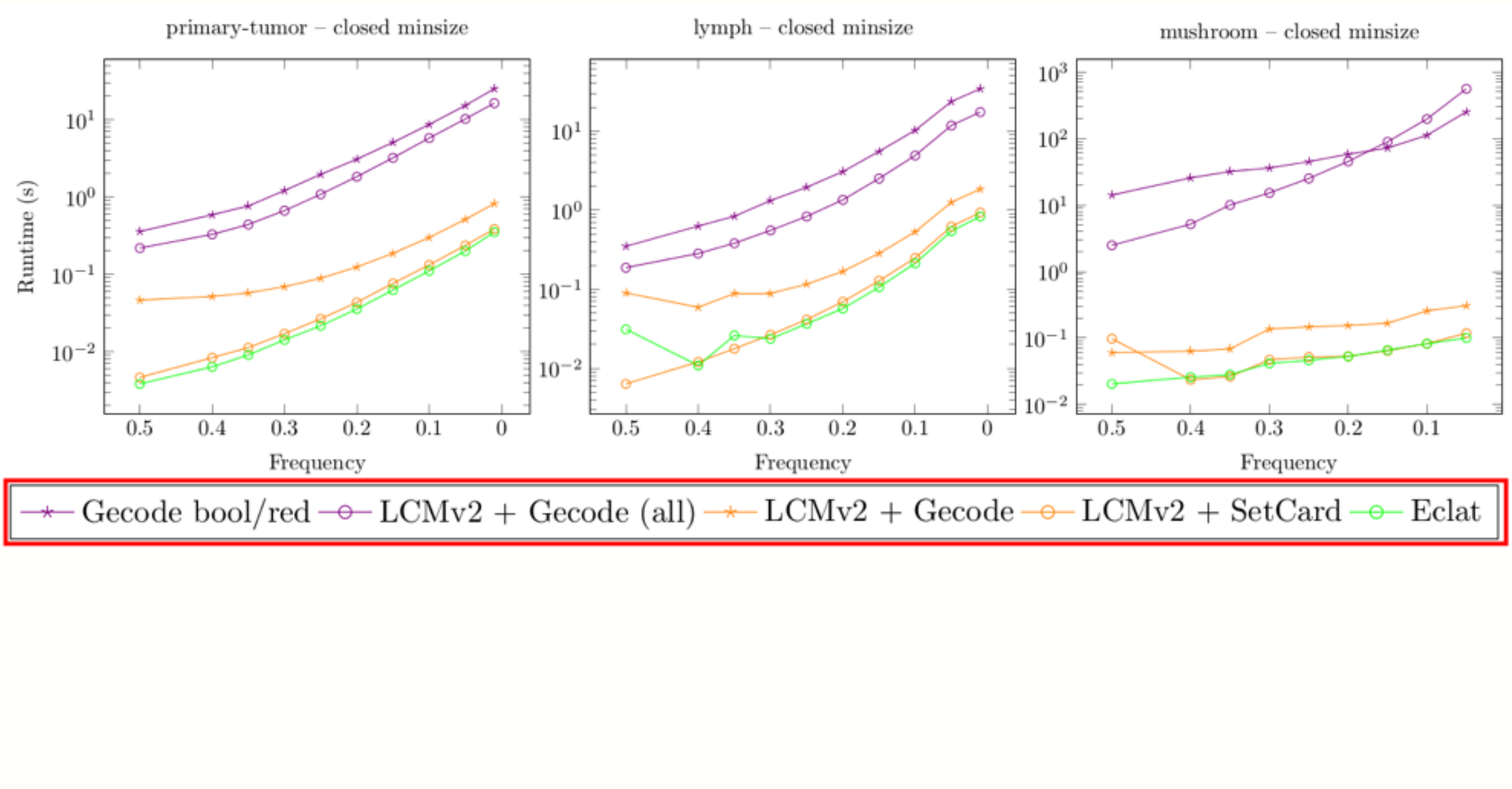
- LCMv2(TDB, 20) + gecode(card(cover(Items, TDB)) =< 40)
- LCMv2(TDB, 20) + frequency(Items, TDB, S) + gecode(S =< 40)

C) Generic solvers only

- gecode(...)
- gecode-bool(...)
- gecode-bool(... *+ redundant*)
- or-tools-bool(...)

# Experiments, hybrid solving

frequent itemset mining, with minimum size and closure constraint



https://dtai.cs.kuleuven.be/CP4IM/download.php

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CP4IM

# Constraint Programming for Itemset Mining

Home | Software | Datasets | Publications

## Software

**MiningZinc**, this high-level declarate framework for constraint-based mining is available on a separate page.

### Download FIM_CP

The frequent and constraint-based itemset mining system using Constraint Programming.

- Publication: *L. De Raedt, T. Guns, S. Nijssen. Constraint programming for itemset mining, KDD 2008.*
- Publication: *T. Guns, S. Nijssen, L. De Raedt. Itemset mining: A constraint programming perspective, Artificial Intelligence 175(12-13), 2011.*
- **Latest version:** fimcp-2.7.tar.gz
- License: MIT license
- Language: C++ (tested on Linux, known to work under Mac and Windows)
- Latest changes: Release with new, easier, build system: it now automatically downloads, configures and compiles the Gecode CP solver (version 3.7.1).

Installation instructions in the accompagnying README. See online usage instructions and examples.

inductiveconstraints.eu/miningzinc/#

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uring Mobilis | Street Photography | Save to Mendeley | Google Scholar | dupong

MiningZinc | Basic usage | Modeling | Examples | Download

# Basic usage

MiningZinc can be used through two interfaces: as a command line tool and as a Python package.

## From the command line

MiningZinc can be run through its command line interface. There are four modes:

- `list` : analyze the model and list the possible execution plans
- `solve` : solve the model
- `interactive` : combination of previous two modes with a choice menu
- default: solve the model using the first available strategy

The default usage of MiningZinc is:

```
./miningzinc model.mzn data1.dzn data2.dzn -D "Param1=10;"
```

# State of the art

- Many other works
  - Clustering (Ian Davidson, Christel Vrain, Thi-Bich-Hanh Dao, …)
  - Skyline patterns (B. Cremilleux, … ; Negrevergne, … )
  - Other pattern types (L. Sais, ..; Guns, … )
- Challenges
  - Scaling
  - Structured data (graphs, sequences, …)
  - Integration with some other ML/DM techniques
  - Other types of solvers …

# Probabilistic Programming

# World Dynamics

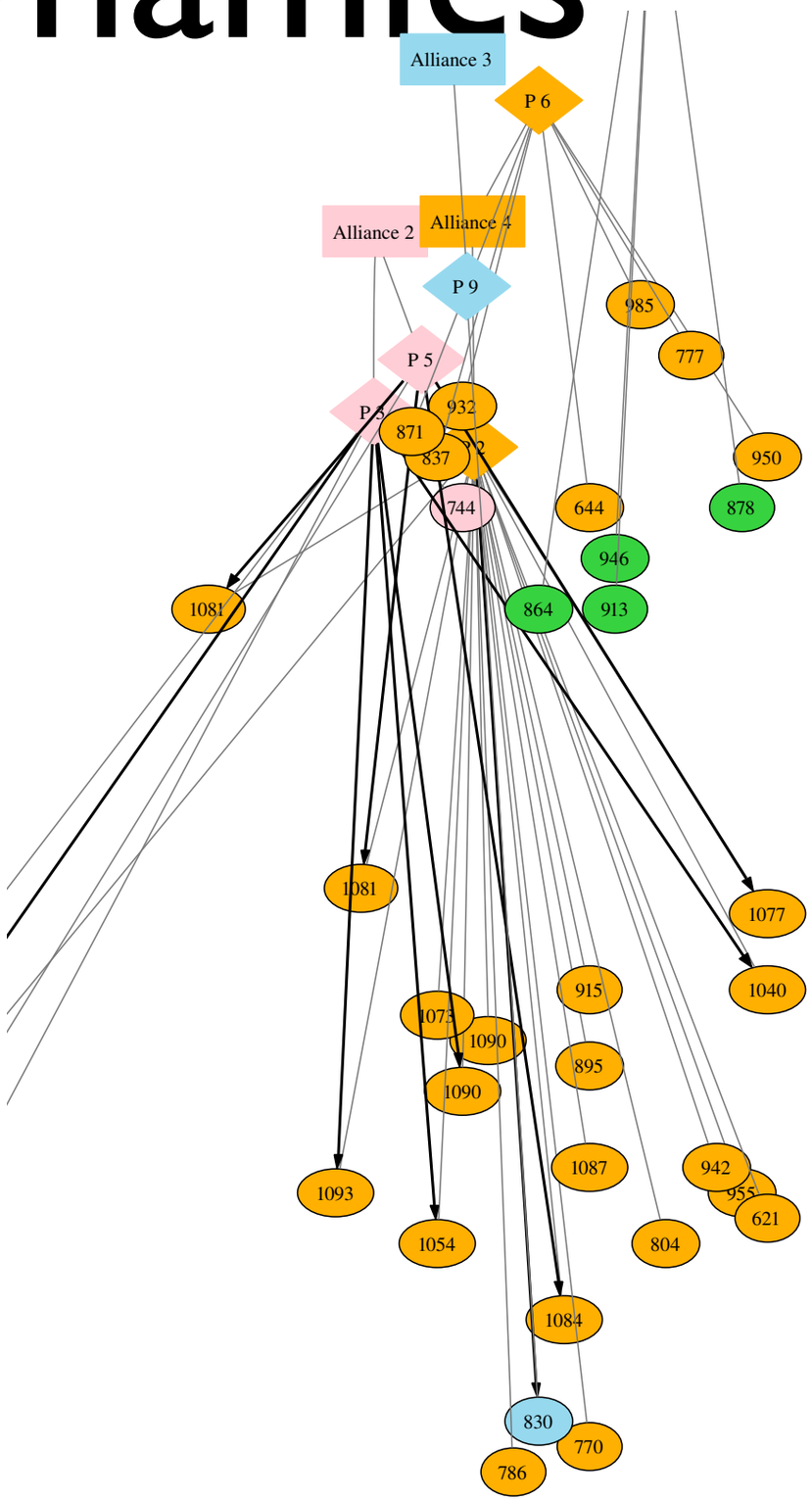
Fragment of world with

~10 alliances
~200 players
~600 cities

alliances color-coded

Can we build a model
of this world ?
Can we use it for playing
better ?

[Thon, Landwehr, De Raedt, ECML08]



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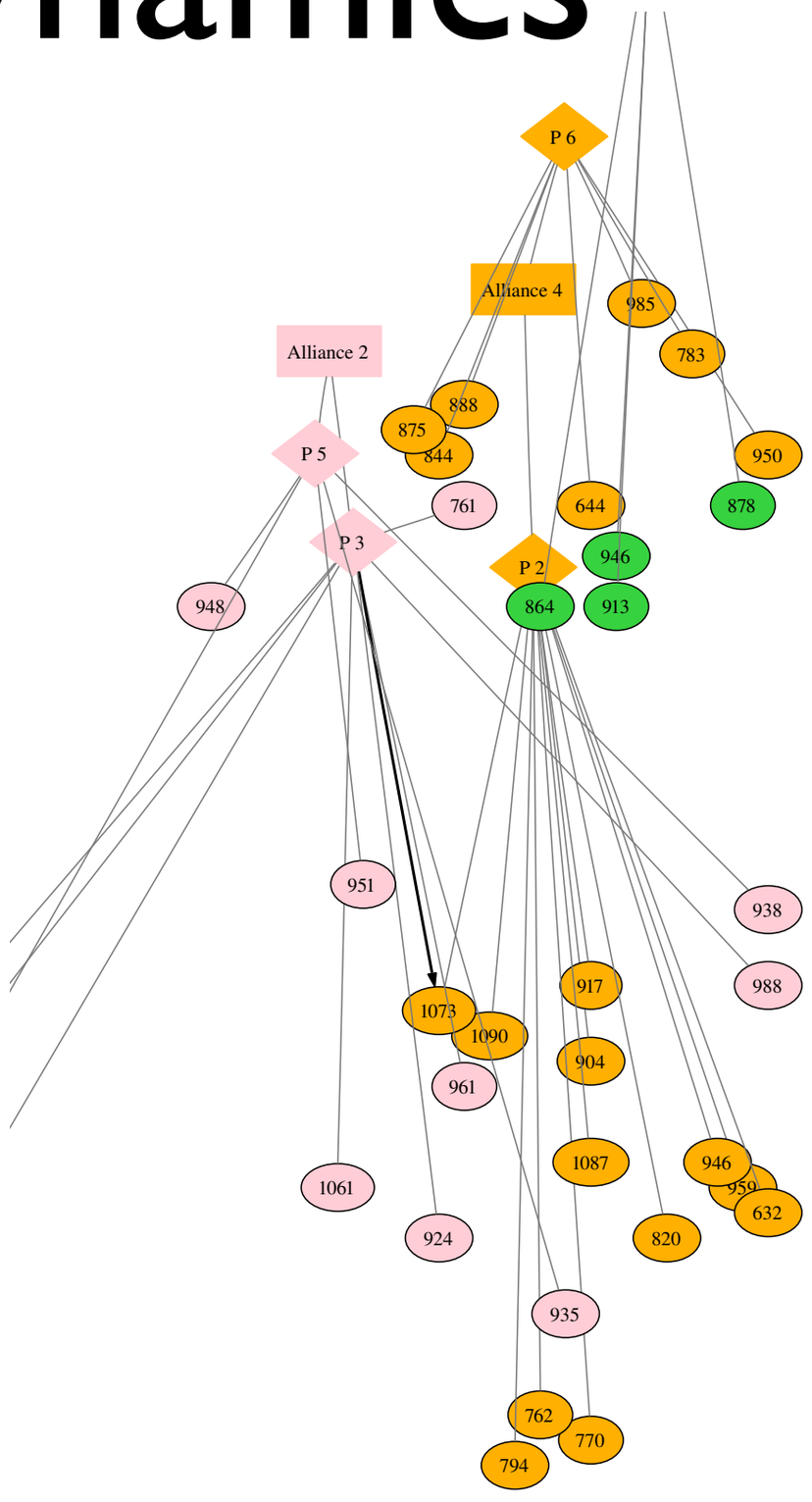

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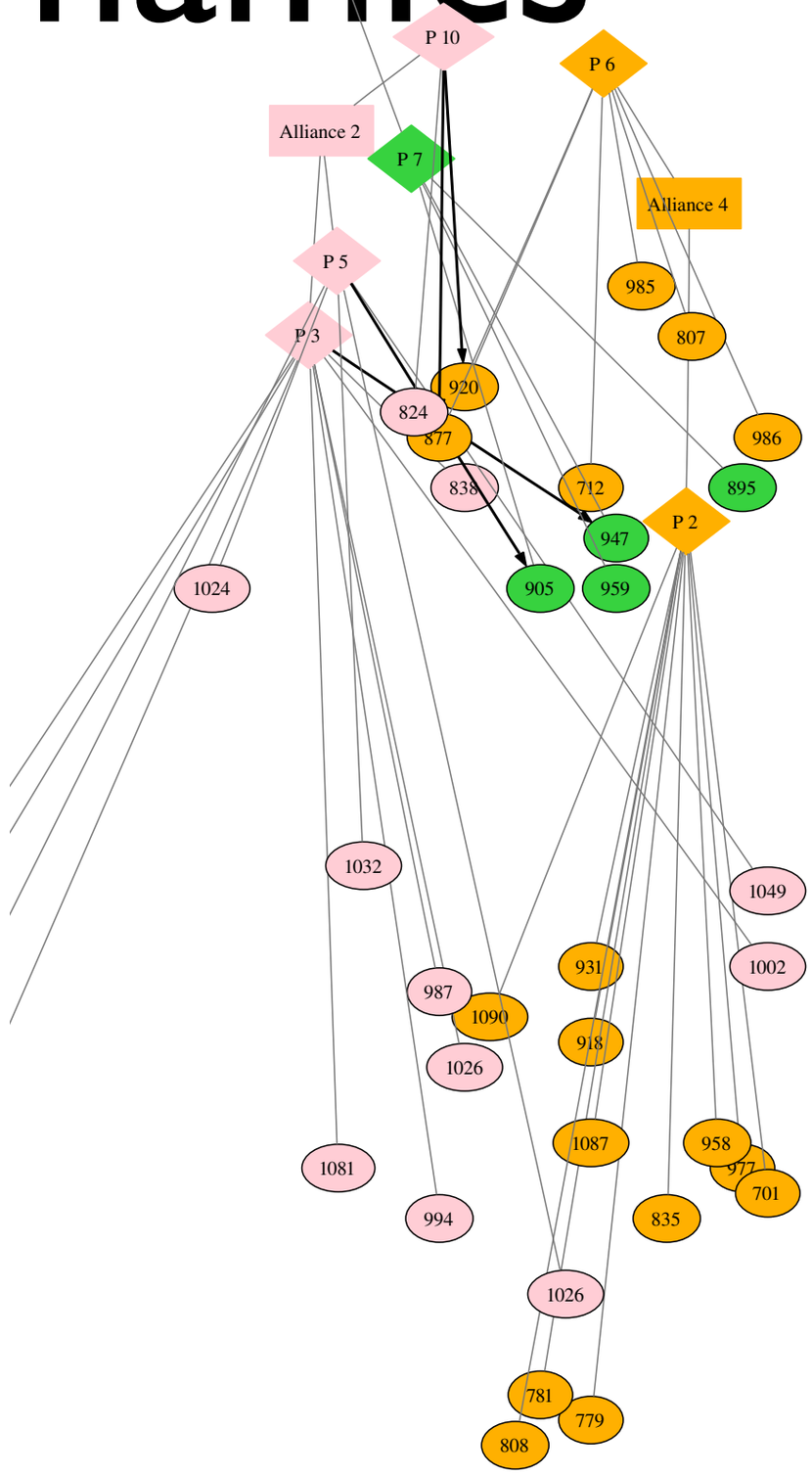
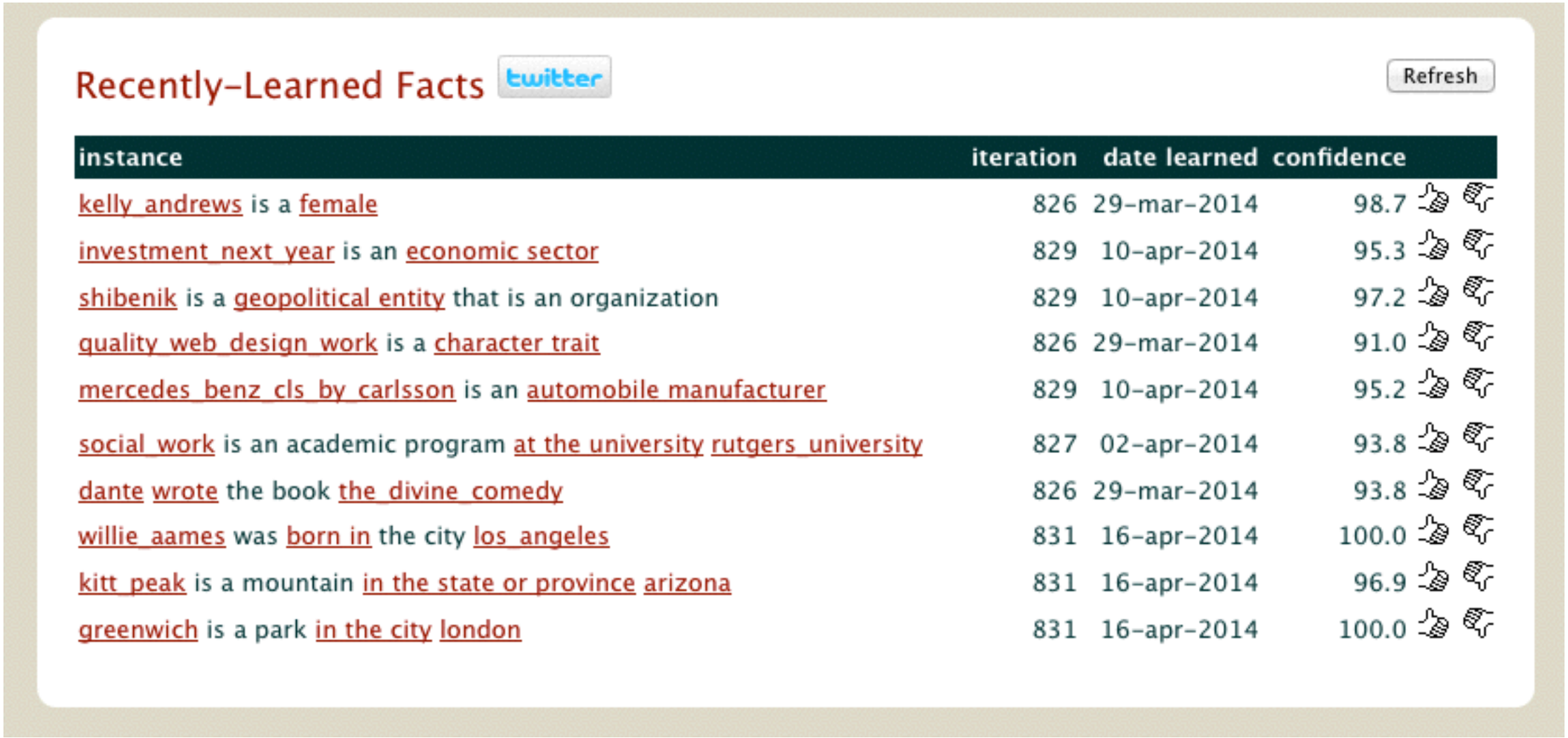


# Example: Information Extraction



Recently-Learned Facts twitter

Refresh

| instance | iteration | date learned | confidence |
|---|---|---|---|
| kelly_andrews is a female | 826 | 29-mar-2014 | 98.7 |
| investment_next_year is an economic sector | 829 | 10-apr-2014 | 95.3 |
| shibenik is a geopolitical entity that is an organization | 829 | 10-apr-2014 | 97.2 |
| quality_web_design_work is a character trait | 826 | 29-mar-2014 | 91.0 |
| mercedes_benz_cls_by_carlsson is an automobile manufacturer | 829 | 10-apr-2014 | 95.2 |
| social_work is an academic program at the university rutgers_university | 827 | 02-apr-2014 | 93.8 |
| dante wrote the book the_divine_comedy | 826 | 29-mar-2014 | 93.8 |
| willie_aames was born in the city los_angeles | 831 | 16-apr-2014 | 100.0 |
| kitt_peak is a mountain in the state or province arizona | 831 | 16-apr-2014 | 96.9 |
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NELL: http://rtw.ml.cmu.edu/rtw/

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instances for many different relations



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instances for many different relations

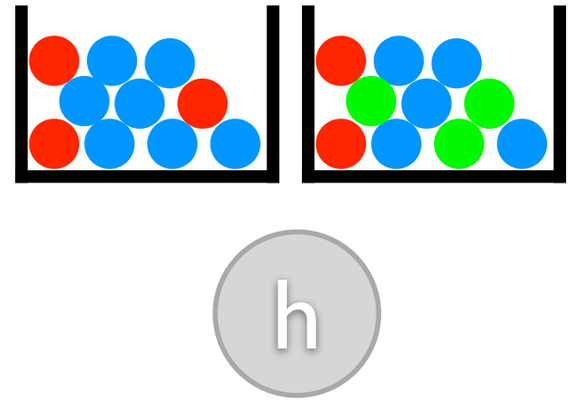
degree of certainty



NELL: http://rtw.ml.cmu.edu/rtw/

ProbLog by example:

# A bit of gambling




- toss (biased) coin & draw ball from each urn
- win if (heads and a red ball) or (two balls of same color)

ProbLog by example:

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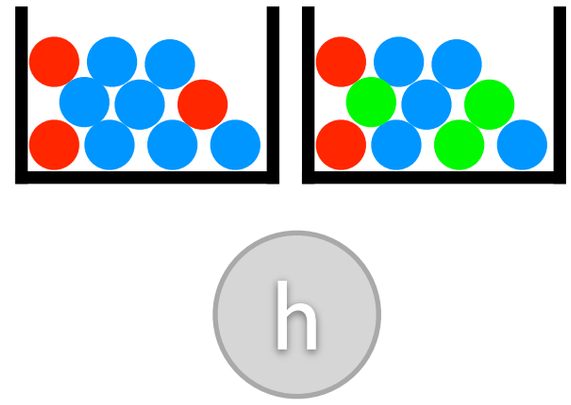


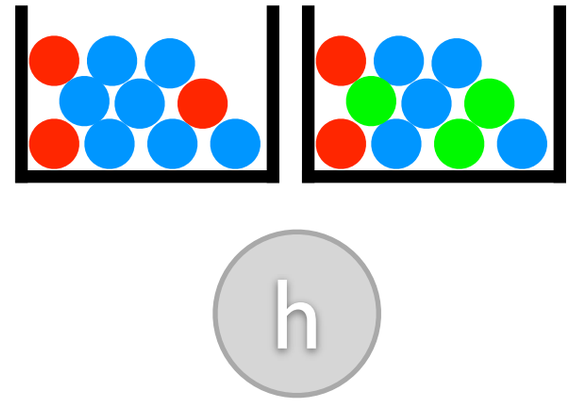

- toss (biased) coin & draw ball from each urn
- win if (heads and a red ball) or (two balls of same color)

```
0.4 :: heads.
```

**probabilistic fact**: heads is true with probability 0.4 (and false with 0.6)

ProbLog by example:

# A bit of gambling




- toss (biased) coin & draw ball from each urn
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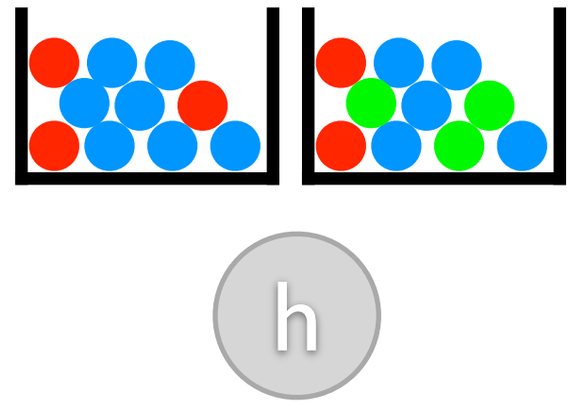
```
0.4 :: heads.

0.3 :: col(1,red); 0.7 :: col(1,blue).
```

**annotated disjunction**: first ball is red with probability 0.3 and blue with 0.7

ProbLog by example:

# A bit of gambling

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- win if (heads and a red ball) or (two balls of same color)

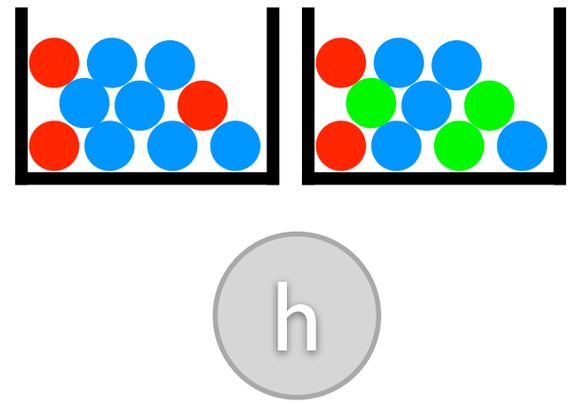
```
0.4 :: heads.

0.3 :: col(1,red); 0.7 :: col(1,blue).
0.2 :: col(2,red); 0.3 :: col(2,green);
                   0.5 :: col(2,blue).
```

**annotated disjunction**: second ball is red with probability 0.2, green with 0.3, and blue with 0.5

ProbLog by example:

# A bit of gambling

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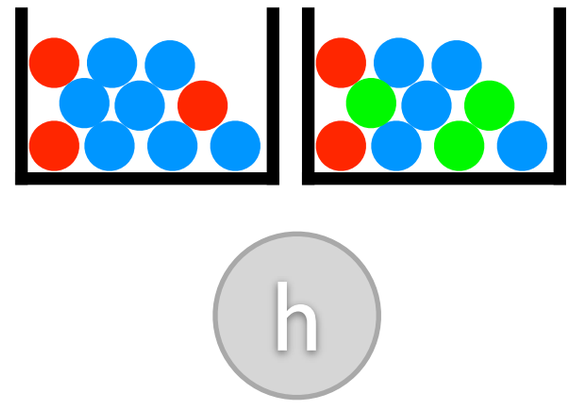
0.2 :: col(2,red); 0.3 :: col(2,green);
                   0.5 :: col(2,blue).

win :- heads, col(_,red).
```

**logical rule** encoding background knowledge

ProbLog by example:

# A bit of gambling




- toss (biased) coin & draw ball from each urn
- win if (heads and a red ball) or (two balls of same color)

```
0.4 :: heads.

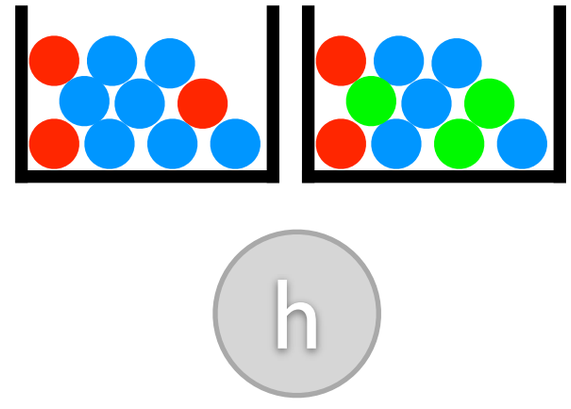
0.3 :: col(1,red); 0.7 :: col(1,blue).
0.2 :: col(2,red); 0.3 :: col(2,green);
                   0.5 :: col(2,blue).

win :- heads, col(_,red).
win :- col(1,C), col(2,C).
```

**logical rule** encoding background knowledge

ProbLog by example:

# A bit of gambling




- toss (biased) coin & draw ball from each urn
- win if (heads and a red ball) or (two balls of same color)

```
0.4 :: heads.

0.3 :: col(1,red); 0.7 :: col(1,blue).
0.2 :: col(2,red); 0.3 :: col(2,green);
                   0.5 :: col(2,blue).
```

**probabilistic choices**

```
win :- heads, col(_,red).
win :- col(1,C), col(2,C).
```

**consequences**

# Questions

```
0.4 :: heads.

0.3 :: col(1,red); 0.7 :: col(1,blue).
0.2 :: col(2,red); 0.3 :: col(2,green); 0.5 :: col(2,blue).

win :- heads, col(_,red).
win :- col(1,C), col(2,C).
```

- Probability of **`win`**?
- Probability of **`win`** given **`col(2,green)`**?
- Most probable world where **`win`** is true?

# Questions

```
0.4 :: heads.

0.3 :: col(1,red); 0.7 :: col(1,blue).
0.2 :: col(2,red); 0.3 :: col(2,green); 0.5 :: col(2,blue).

win :- heads, col(_,red).
win :- col(1,C), col(2,C).
```

**marginal probability**

- Probability of `win`?

**query**

- Probability of `win` given `col(2,green)`?
- Most probable world where `win` is true?

# Questions

```
0.4 :: heads.

0.3 :: col(1,red); 0.7 :: col(1,blue).
0.2 :: col(2,red); 0.3 :: col(2,green); 0.5 :: col(2,blue).

win :- heads, col(_,red).
win :- col(1,C), col(2,C).
```

**marginal probability**

- Probability of **`win`**?

**conditional probability**

- Probability of **`win`** given **`col(2,green)`**?

**evidence**

- Most probable world where **`win`** is true?

# Questions

```
0.4 :: heads.

0.3 :: col(1,red); 0.7 :: col(1,blue).
0.2 :: col(2,red); 0.3 :: col(2,green); 0.5 :: col(2,blue).

win :- heads, col(_,red).
win :- col(1,C), col(2,C).
```

**marginal probability**

- Probability of **`win`**?

**conditional probability**

- Probability of **`win`** given **`col(2,green)`**?

- Most probable world where **`win`** is true?

**MPE inference**

# Possible Worlds

```
0.4 :: heads.

0.3 :: col(1,red); 0.7 :: col(1,blue).
0.2 :: col(2,red); 0.3 :: col(2,green); 0.5 :: col(2,blue).

win :- heads, col( ,red).
win :- col(1,C), col(2,C).
```

# Possible Worlds

```
0.4 :: heads.

0.3 :: col(1,red); 0.7 :: col(1,blue).
0.2 :: col(2,red); 0.3 :: col(2,green); 0.5 :: col(2,blue).

win :- heads, col( ,red).
win :- col(1,C), col(2,C).
```

# Possible Worlds

```
0.4 :: heads.

0.3 :: col(1,red); 0.7 :: col(1,blue).
0.2 :: col(2,red); 0.3 :: col(2,green); 0.5 :: col(2,blue).

win :- heads, col(_,red).
win :- col(1,C), col(2,C).
```

0.4

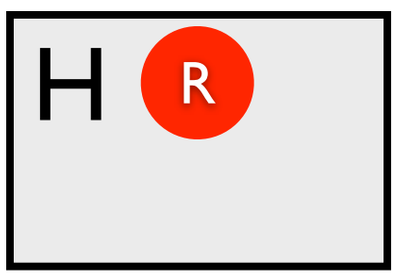
# Possible Worlds

```
0.4 :: heads.

0.3 :: col(1,red); 0.7 :: col(1,blue).
0.2 :: col(2,red); 0.3 :: col(2,green); 0.5 :: col(2,blue).

win :- heads, col(_,red).
win :- col(1,C), col(2,C).
```

0.4 ×0.3



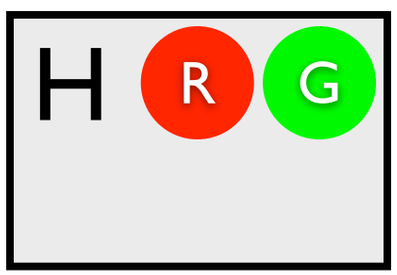
# Possible Worlds

```
0.4 :: heads.

0.3 :: col(1,red); 0.7 :: col(1,blue).
0.2 :: col(2,red); 0.3 :: col(2,green); 0.5 :: col(2,blue).

win :- heads, col(_,red).
win :- col(1,C), col(2,C).
```

0.4 ×0.3 ×0.3



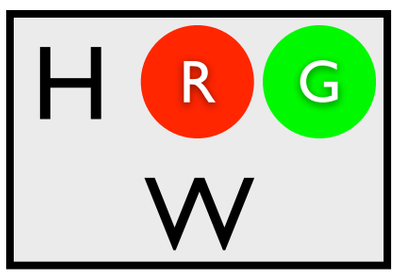
# Possible Worlds

```
0.4 :: heads.

0.3 :: col(1,red); 0.7 :: col(1,blue).
0.2 :: col(2,red); 0.3 :: col(2,green); 0.5 :: col(2,blue).

win :- heads, col(_,red).
win :- col(1,C), col(2,C).
```

0.4 ×0.3 ×0.3



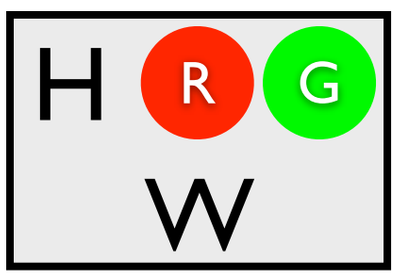
# Possible Worlds

```
0.4 :: heads.

0.3 :: col(1,red); 0.7 :: col(1,blue).
0.2 :: col(2,red); 0.3 :: col(2,green); 0.5 :: col(2,blue).

win :- heads, col( ,red).
win :- col(1,C), col(2,C).
```

$0.4 \times 0.3 \times 0.3$




$(1-0.4)$

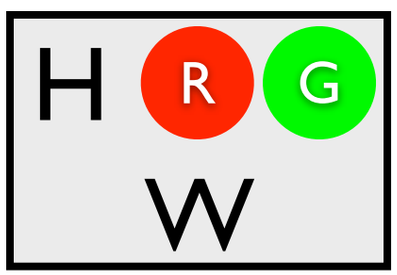
# Possible Worlds

```
0.4 :: heads.

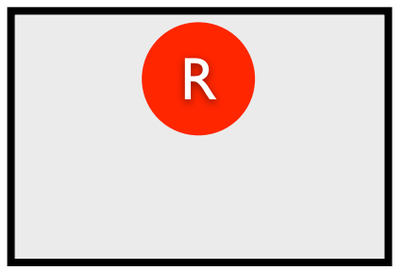
0.3 :: col(1,red); 0.7 :: col(1,blue).
0.2 :: col(2,red); 0.3 :: col(2,green); 0.5 :: col(2,blue).

win :- heads, col( ,red).
win :- col(1,C), col(2,C).
```

$0.4 \times 0.3 \times 0.3$




$(1-0.4) \times 0.3$



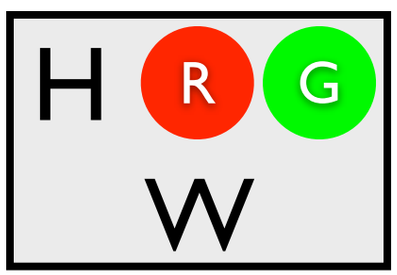
# Possible Worlds

```
0.4 :: heads.

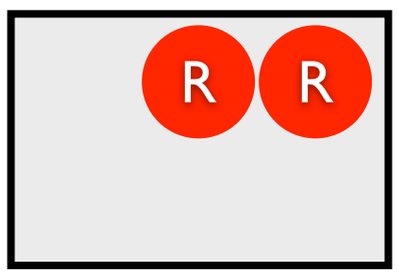
0.3 :: col(1,red); 0.7 :: col(1,blue).
0.2 :: col(2,red); 0.3 :: col(2,green); 0.5 :: col(2,blue).

win :- heads, col(_,red).
win :- col(1,C), col(2,C).
```

0.4 ×0.3 ×0.3




(1−0.4)×0.3 ×0.2



# Possible Worlds

```
0.4 :: heads.

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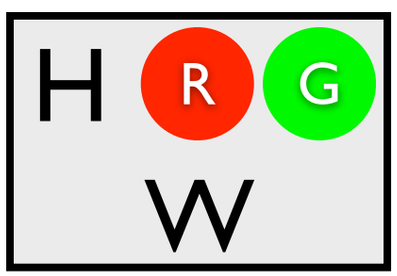
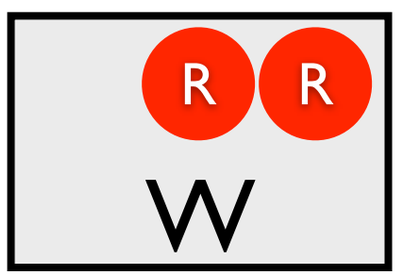
0.2 :: col(2,red); 0.3 :: col(2,green); 0.5 :: col(2,blue).

win :- heads, col(_,red).
win :- col(1,C), col(2,C).
```

$0.4 \times 0.3 \times 0.3$




$(1-0.4) \times 0.3 \times 0.2$



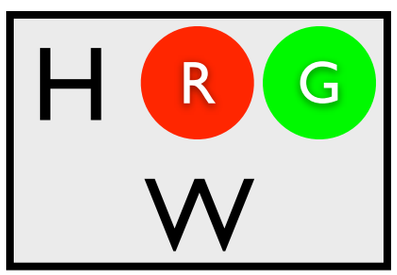
# Possible Worlds

```
0.4 :: heads.

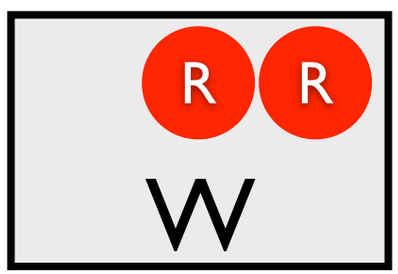
0.3 :: col(1,red); 0.7 :: col(1,blue).
0.2 :: col(2,red); 0.3 :: col(2,green); 0.5 :: col(2,blue).

win :- heads, col(_,red).
win :- col(1,C), col(2,C).
```

$0.4 \times 0.3 \times 0.3$




$(1-0.4) \times 0.3 \times 0.2$




$(1-0.4)$

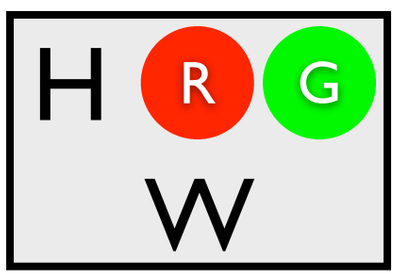
# Possible Worlds

```
0.4 :: heads.

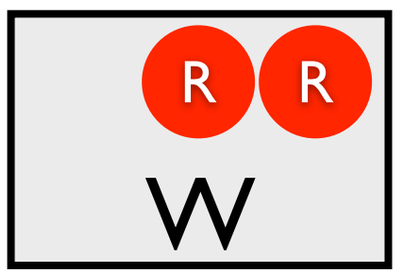
0.3 :: col(1,red); 0.7 :: col(1,blue).
0.2 :: col(2,red); 0.3 :: col(2,green); 0.5 :: col(2,blue).

win :- heads, col( ,red).
win :- col(1,C), col(2,C).
```

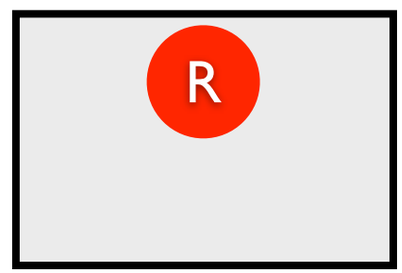
0.4 ×0.3 ×0.3




(1−0.4)×0.3 ×0.2




(1−0.4)×0.3



# Possible Worlds

```
0.4 :: heads.

0.3 :: col(1,red); 0.7 :: col(1,blue).
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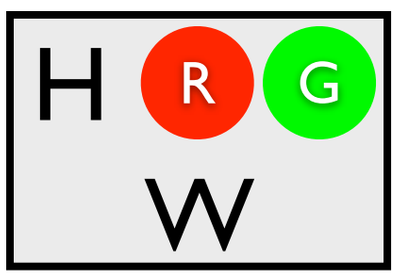
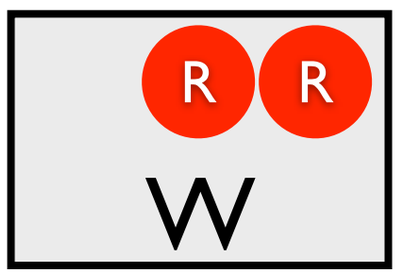
```

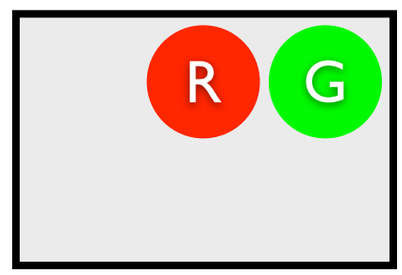
0.4 ×0.3 ×0.3




(1−0.4)×0.3 ×0.2




(1−0.4)×0.3 ×0.3



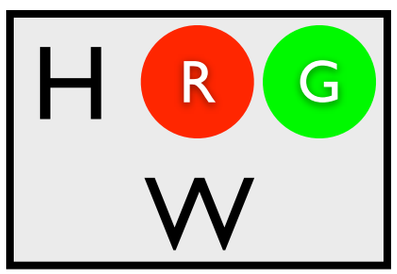
# Possible Worlds

```
0.4 :: heads.

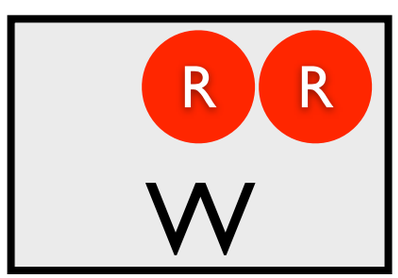
0.3 :: col(1,red); 0.7 :: col(1,blue).
0.2 :: col(2,red); 0.3 :: col(2,green); 0.5 :: col(2,blue).

win :- heads, col(_,red).
win :- col(1,C), col(2,C).
```

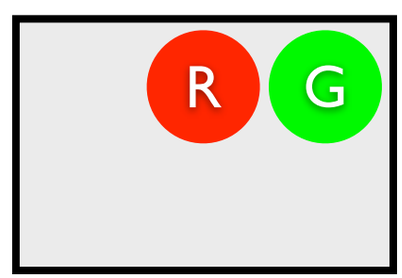
$0.4 \times 0.3 \times 0.3$




$(1-0.4) \times 0.3 \times 0.2$




$(1-0.4) \times 0.3 \times 0.3$



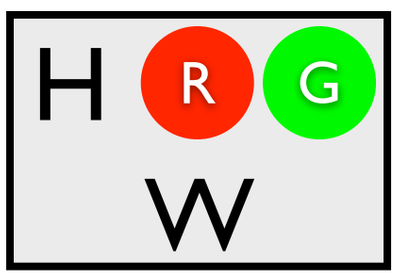
# Possible Worlds

```
0.4 :: heads.

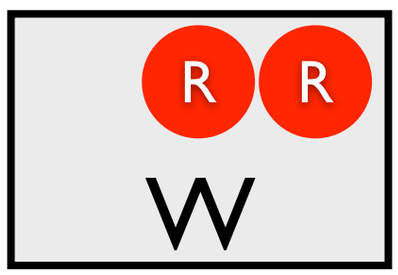
0.3 :: col(1,red); 0.7 :: col(1,blue).
0.2 :: col(2,red); 0.3 :: col(2,green); 0.5 :: col(2,blue).

win :- heads, col( ,red).
win :- col(1,C), col(2,C).
```

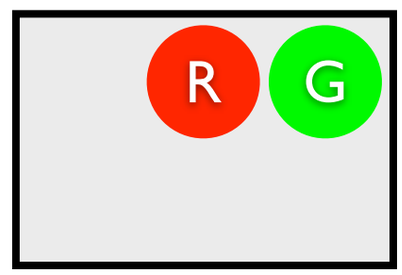
0.4 ×0.3 ×0.3




(1−0.4)×0.3 ×0.2

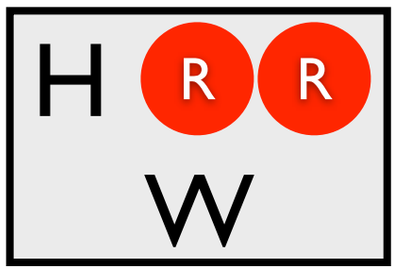



(1−0.4)×0.3 ×0.3

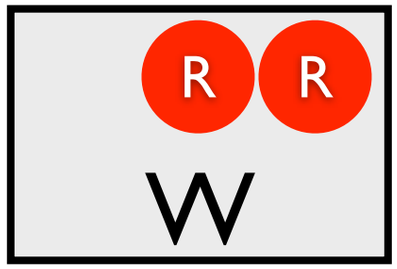


# All Possible Worlds

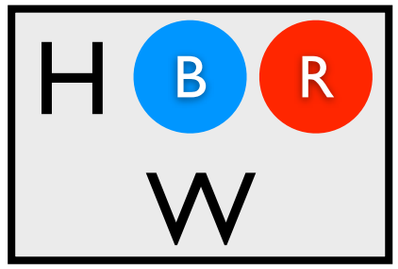
0.024



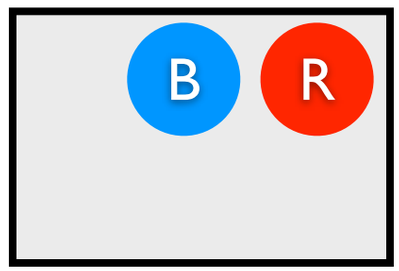

0.036




0.056




0.084

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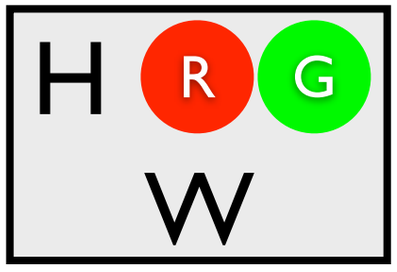
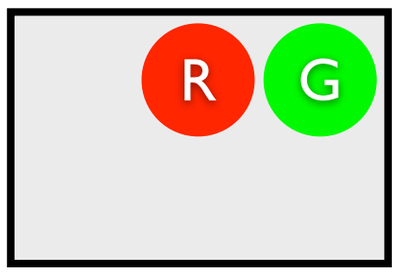
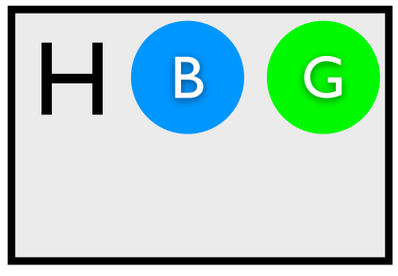


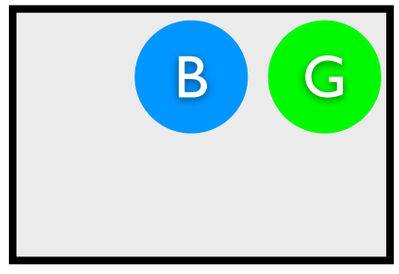

0.054



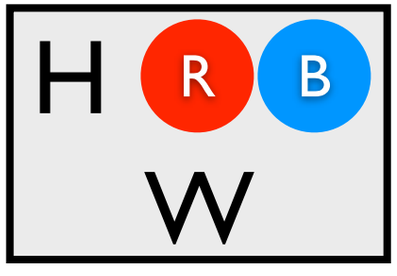

0.084



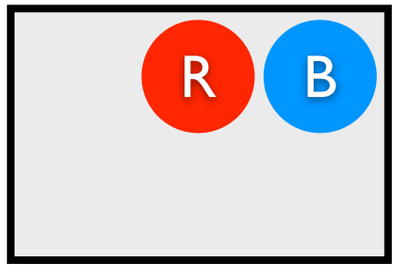

0.126



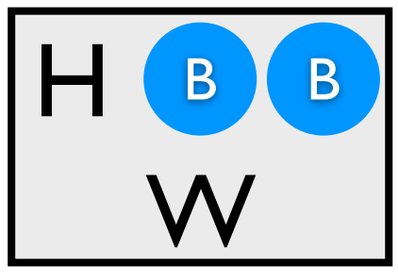

0.060



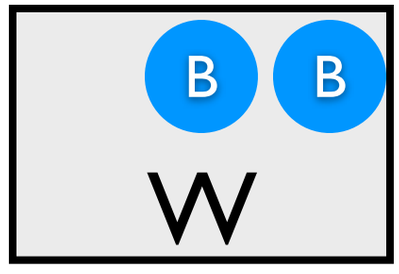

0.090




0.140

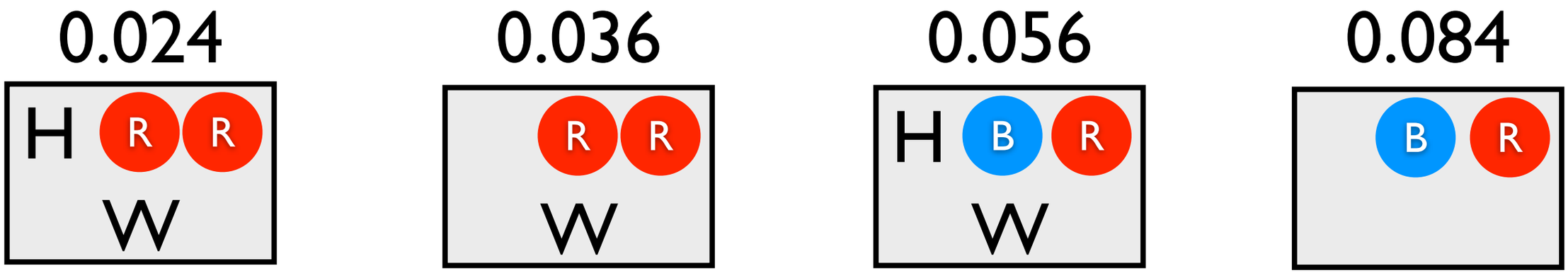



0.210



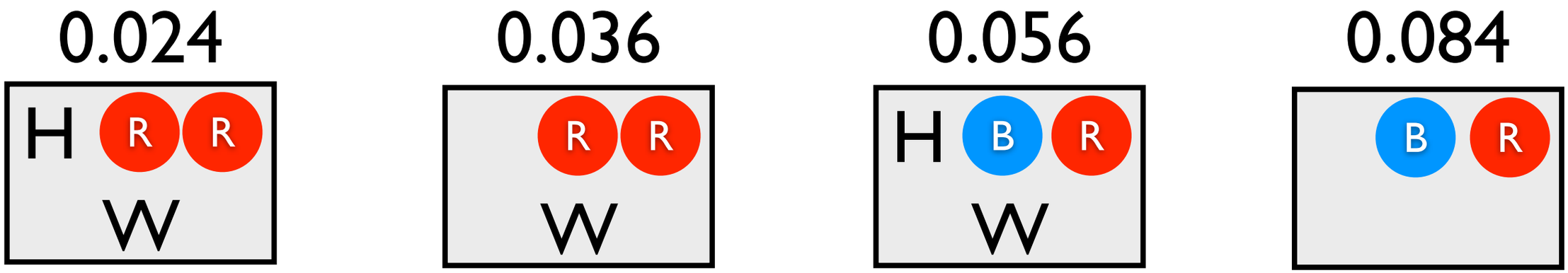
# Most likely world where **`win`** is true?

MPE Inference



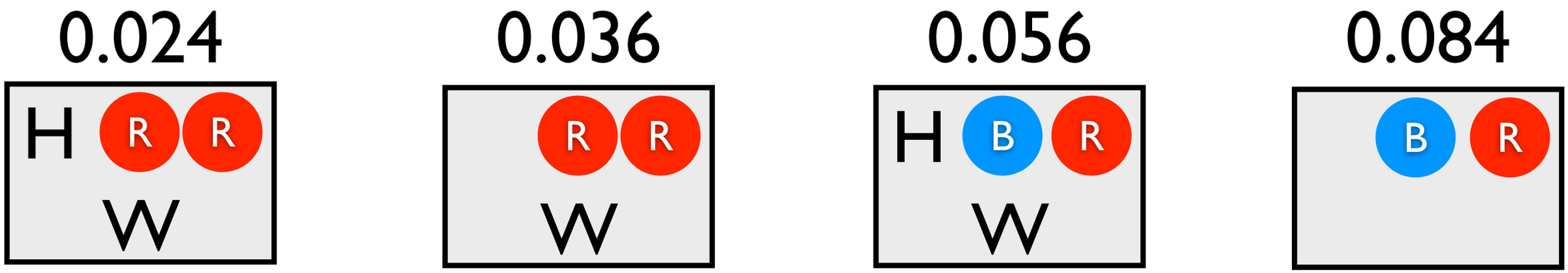
# Most likely world where **`win`** is true?

MPE Inference



# Most likely world where `col(2,blue)` is false?

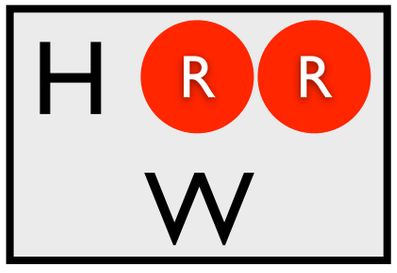
MPE Inference



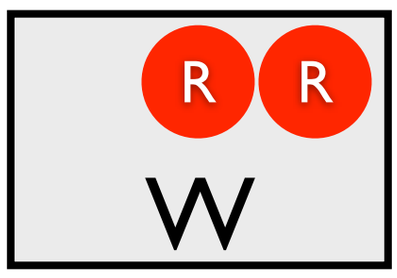
# Most likely world where `col(2,blue)` is false?

MPE Inference

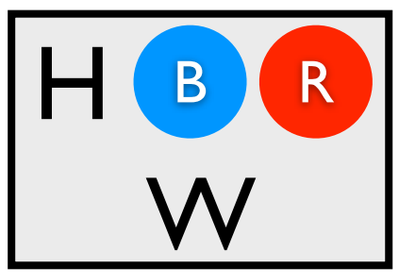
0.024



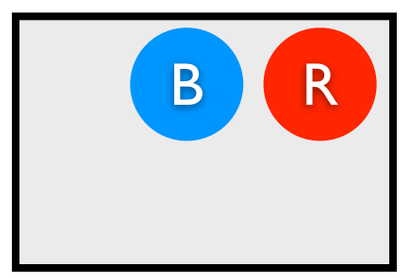

0.036




0.056




0.084

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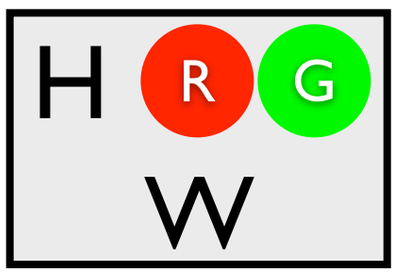
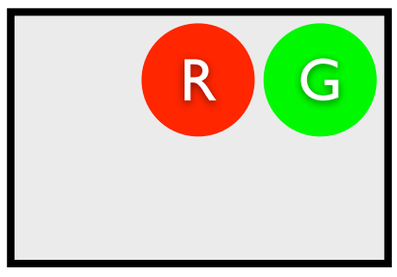



0.054

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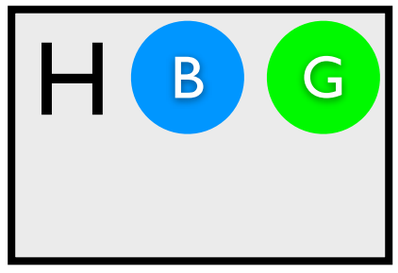
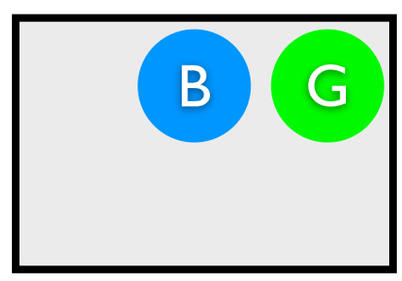



0.126

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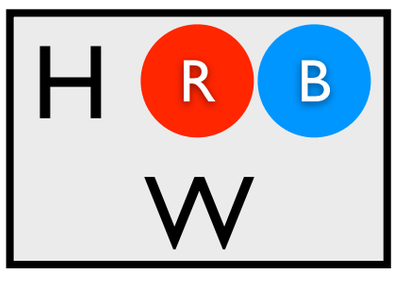


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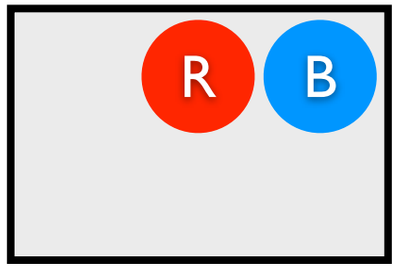
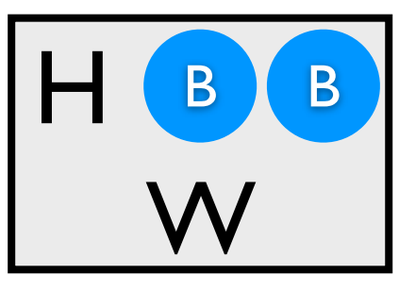
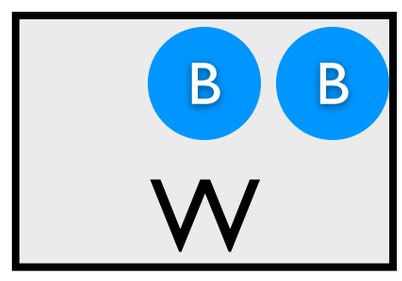



0.140

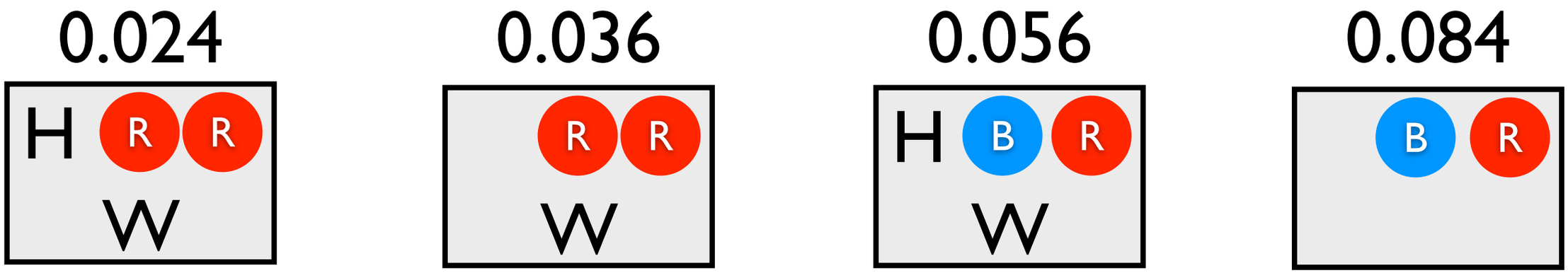



0.210



# P(win)=?

Marginal Probability

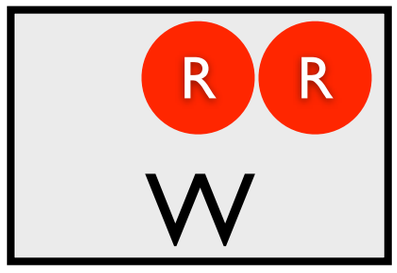


# P(win)= Σ

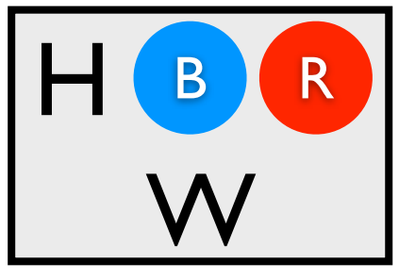
Marginal Probability

0.024

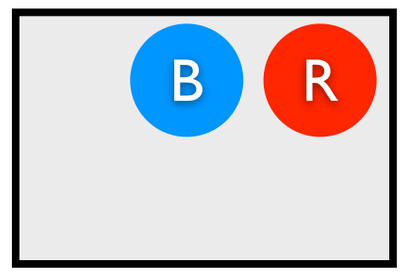

0.036



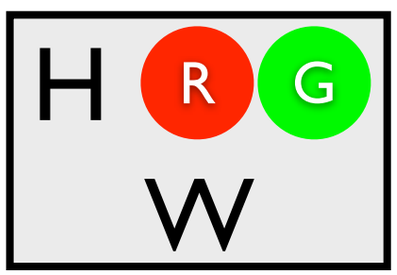

0.056



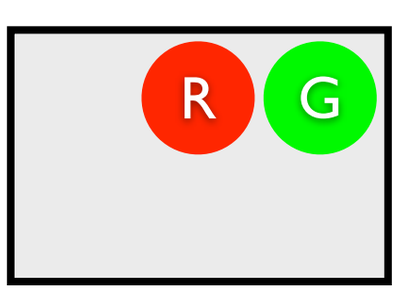

0.084



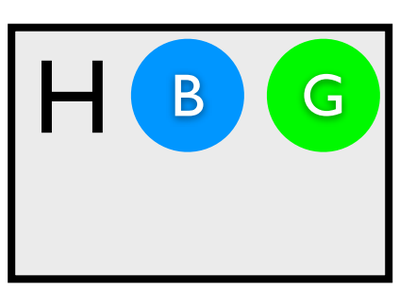

0.036



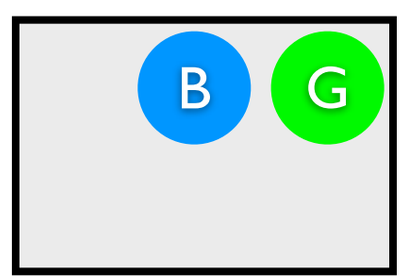

0.054



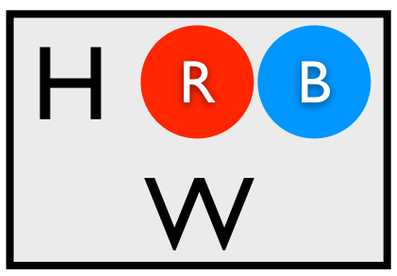

0.084



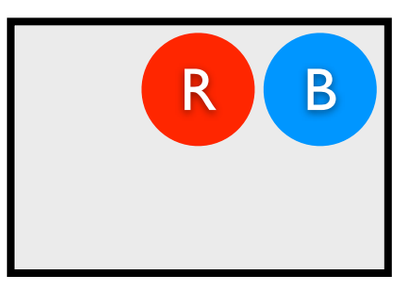

0.126



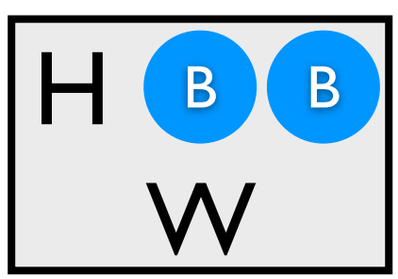

0.060



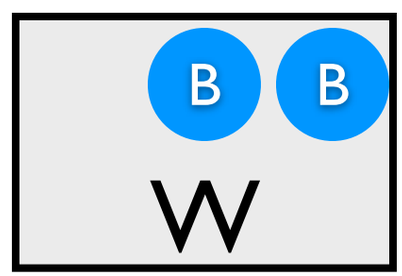

0.090




0.140




0.210

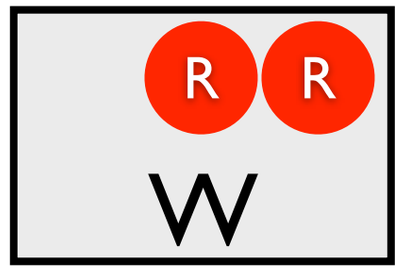


# P(win)= Σ =0.562

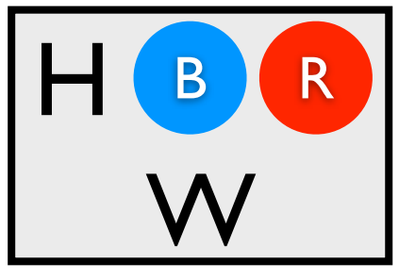
Marginal Probability

0.024


0.036




0.056

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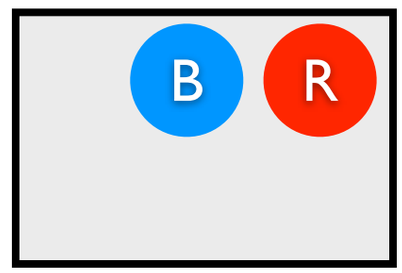
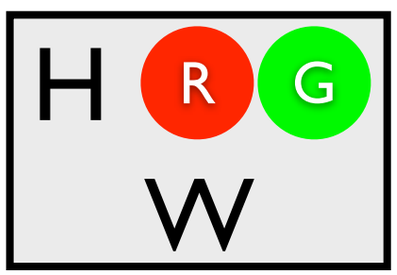
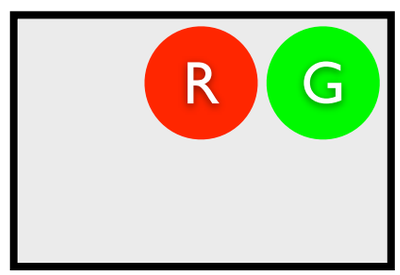



0.036




0.054

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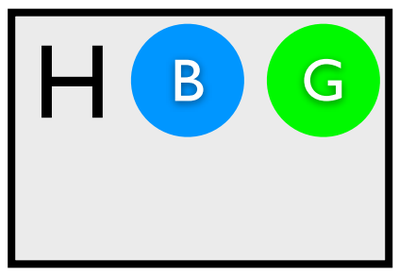
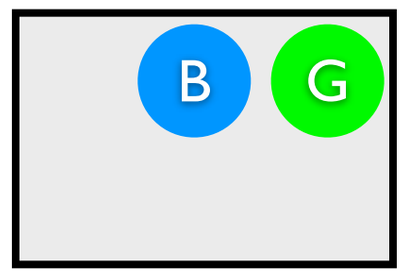
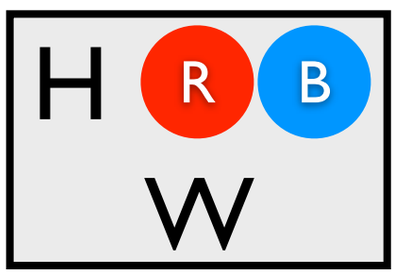


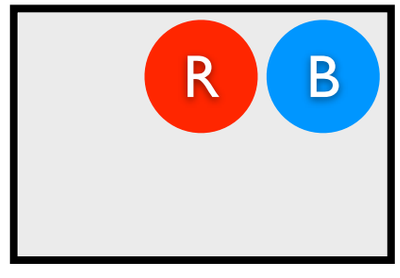

0.126



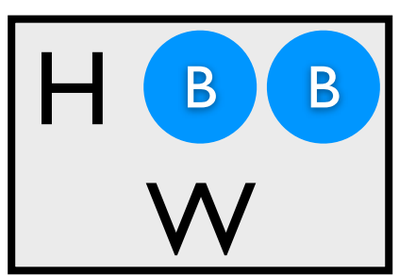

0.060



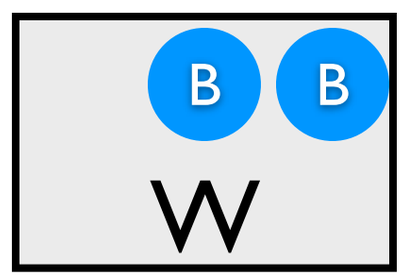

0.090




0.140



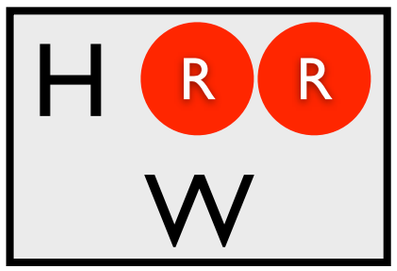

0.210



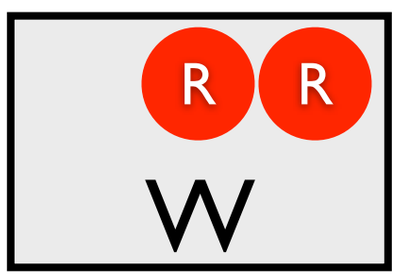
P(win|col(2,green))= ?

Conditional Probability

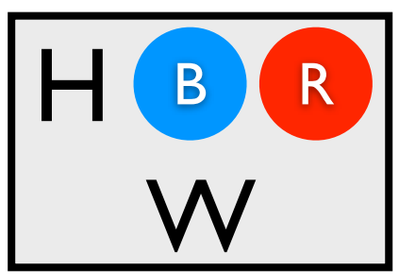
0.024



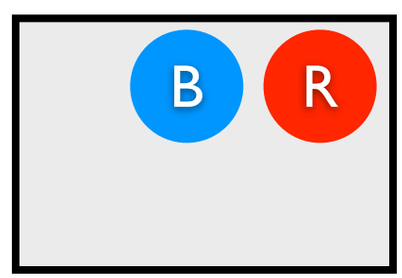

0.036




0.056




0.084

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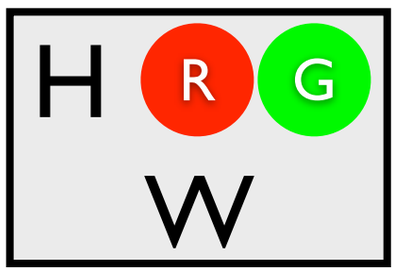


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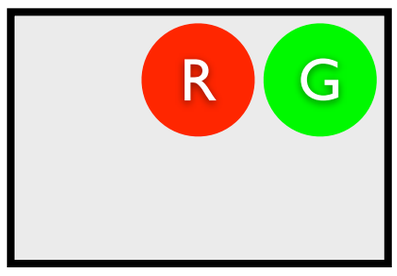


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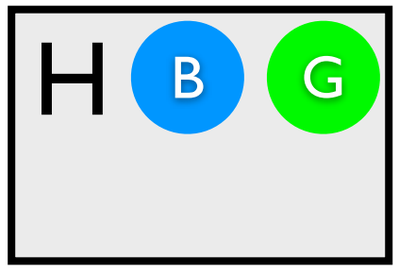
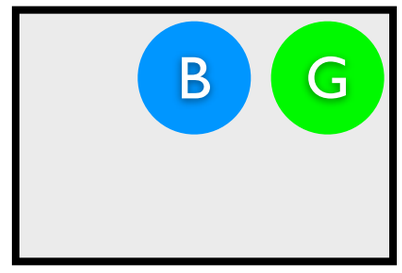



0.126

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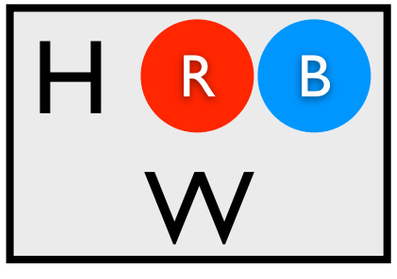


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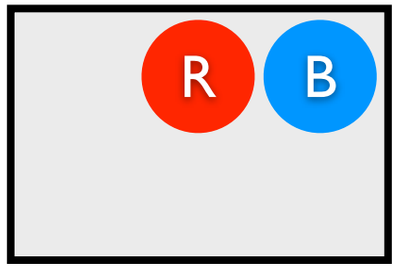


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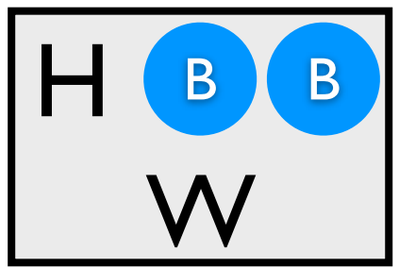


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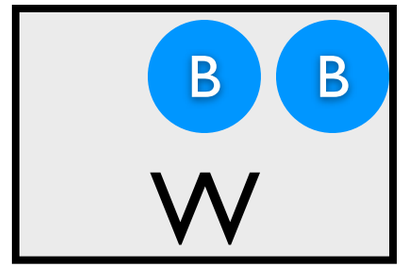
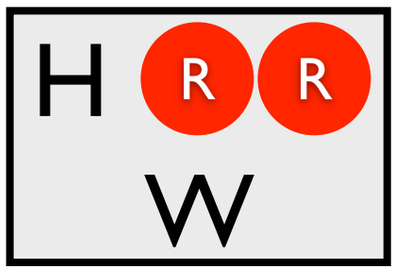


$$P(win|col(2,green)) = \Sigma/\Sigma$$

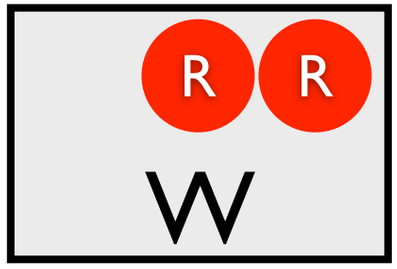
$$= P(win \wedge col(2,green))/P(col(2,green))$$

Conditional Probability

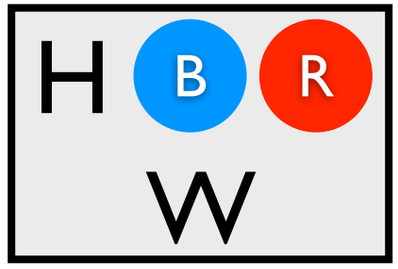
0.024



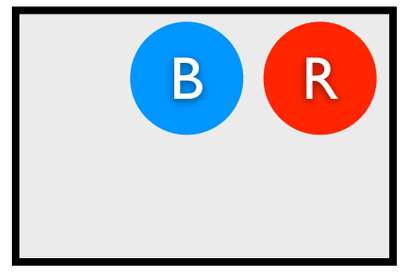

0.036



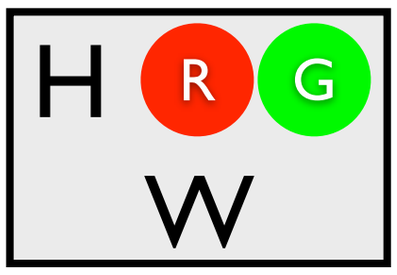

0.056



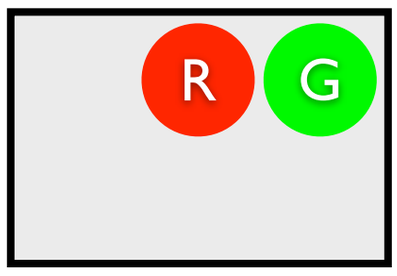

0.084



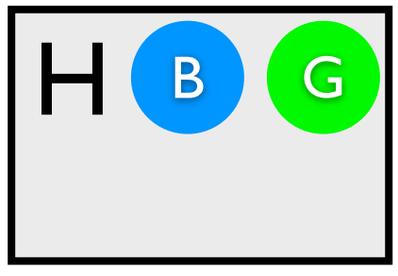

0.036



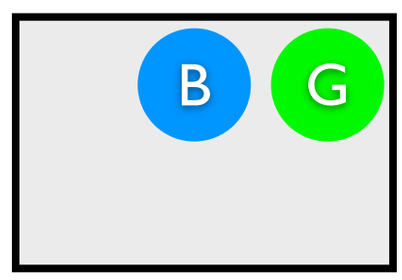

0.054



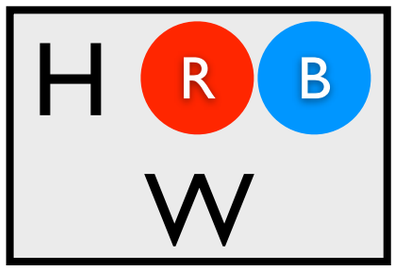

0.084



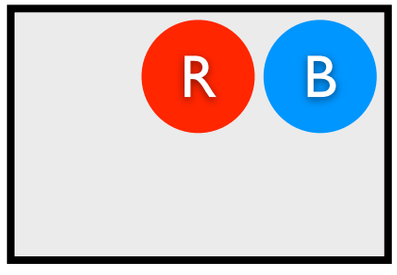

0.126



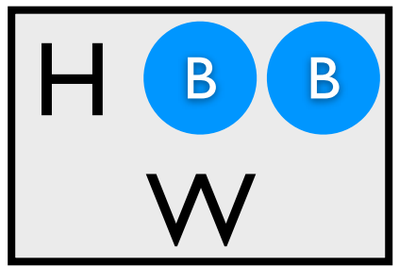

0.060



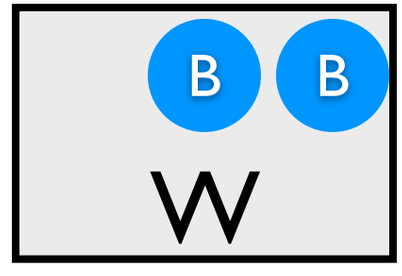

0.090




0.140



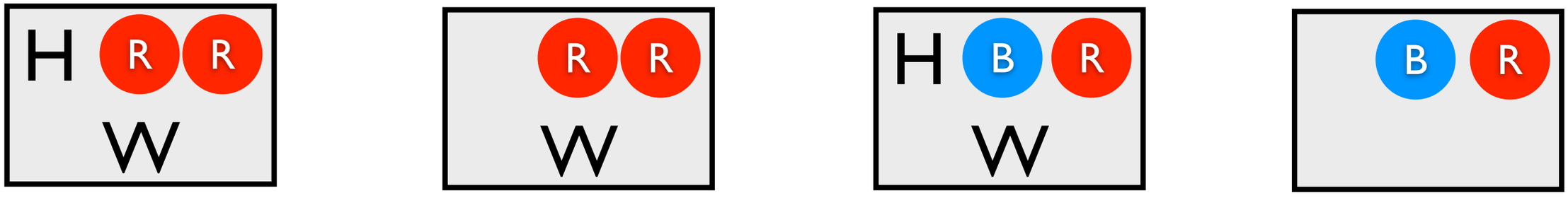

0.210

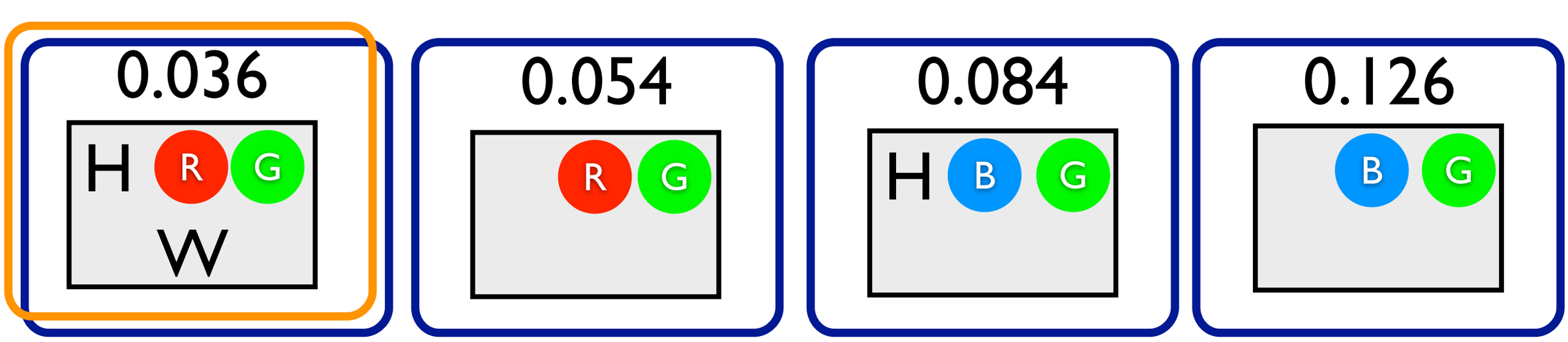


$$P(win|col(2,green)) = \Sigma/\Sigma$$

$$= P(win \wedge col(2,green))/P(col(2,green))$$

Conditional Probability

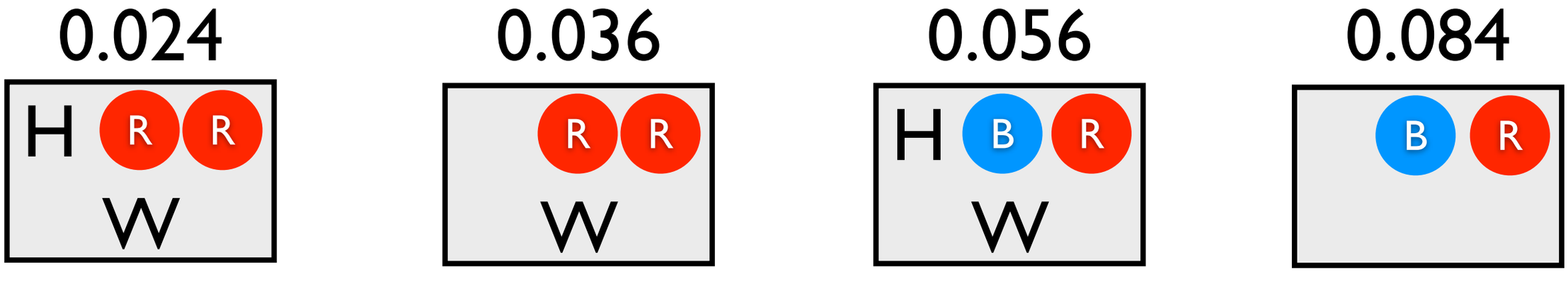


$$P(win|col(2,green)) = \Sigma/\Sigma$$

$$= 0.036/0.3 = 0.12$$

Conditional Probability



# Inference in PLP

- As in Prolog and logic programming
  - proof-based
- As in Answer Set Programming
  - model based
- As in Probabilistic Programming
  - sampling

# Parameter Learning

e.g., webpage classification model

for each *CLASS1*, *CLASS2* and each *WORD*

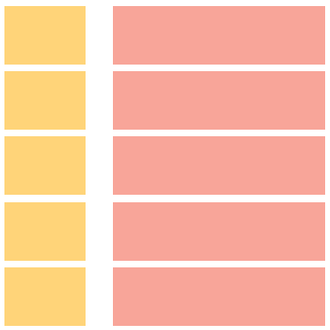
?? :: link_class(Source,Target,*CLASS1*,*CLASS2*).
?? :: word_class(*WORD*,*CLASS*).

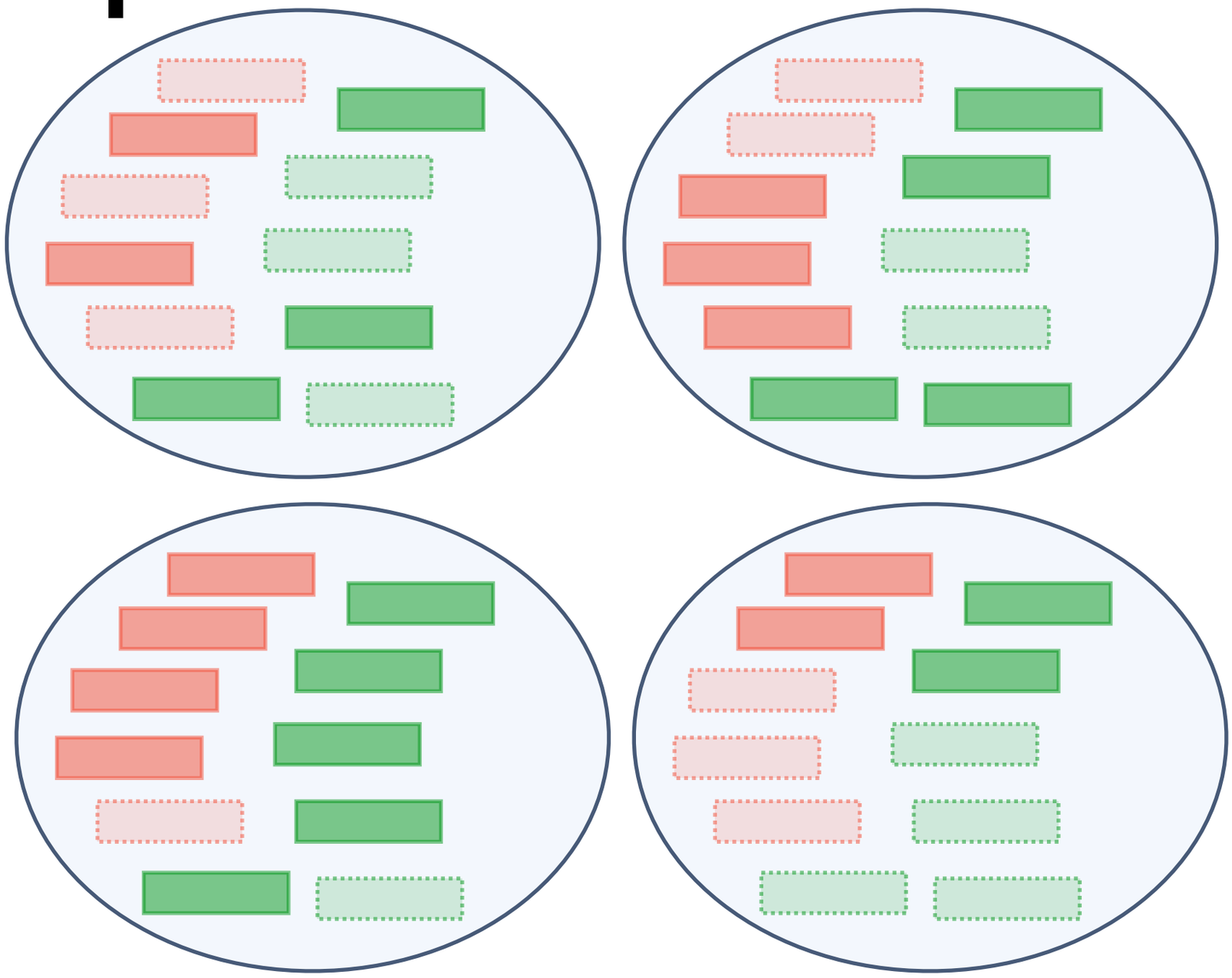
```
class(Page,C)    :-  has_word(Page,W), word_class(W,C).

class(Page,C)        :-  links_to(OtherPage,Page),
    class(OtherPage,OtherClass),
                         link_class(OtherPage,Page,OtherClass,C).
```

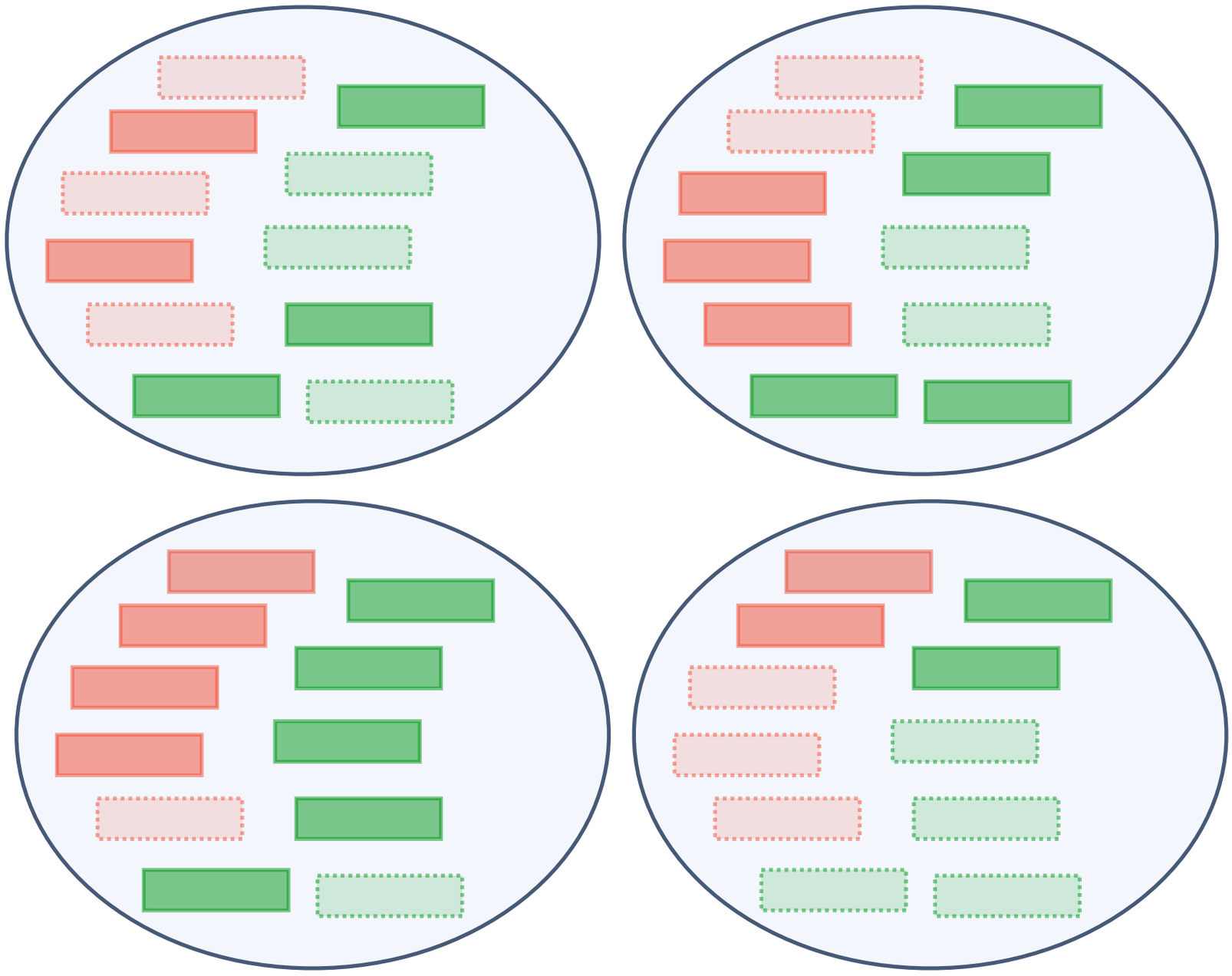
# Sampling Interpretations

# Sampling Interpretations

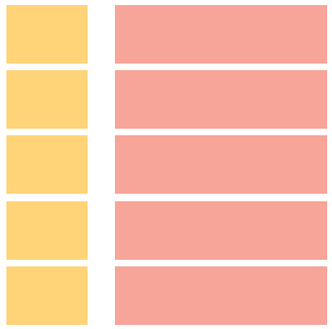


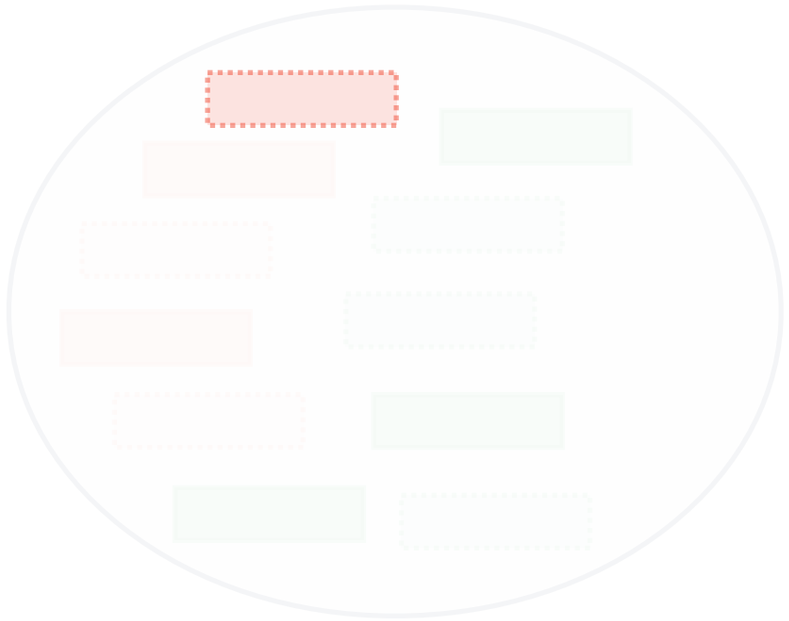


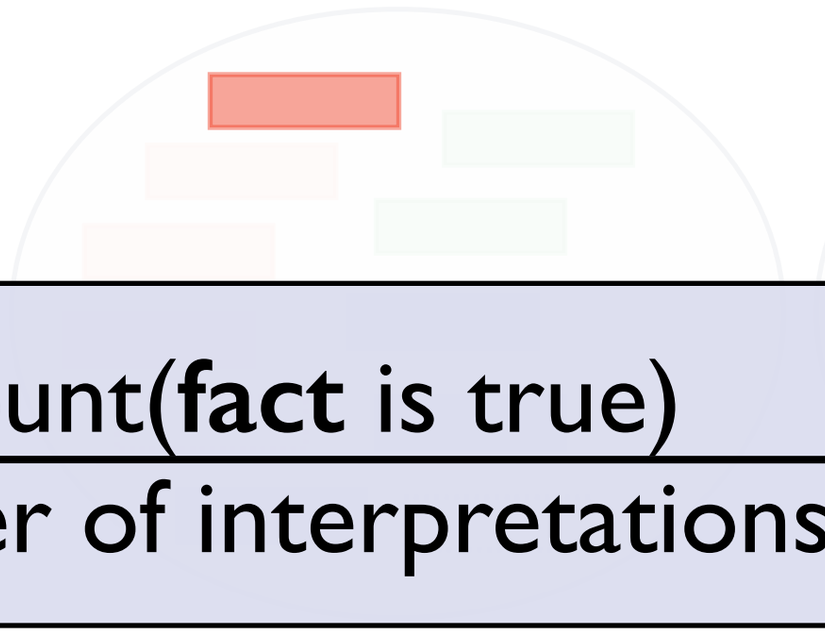
# Parameter Estimation



# Parameter Estimation

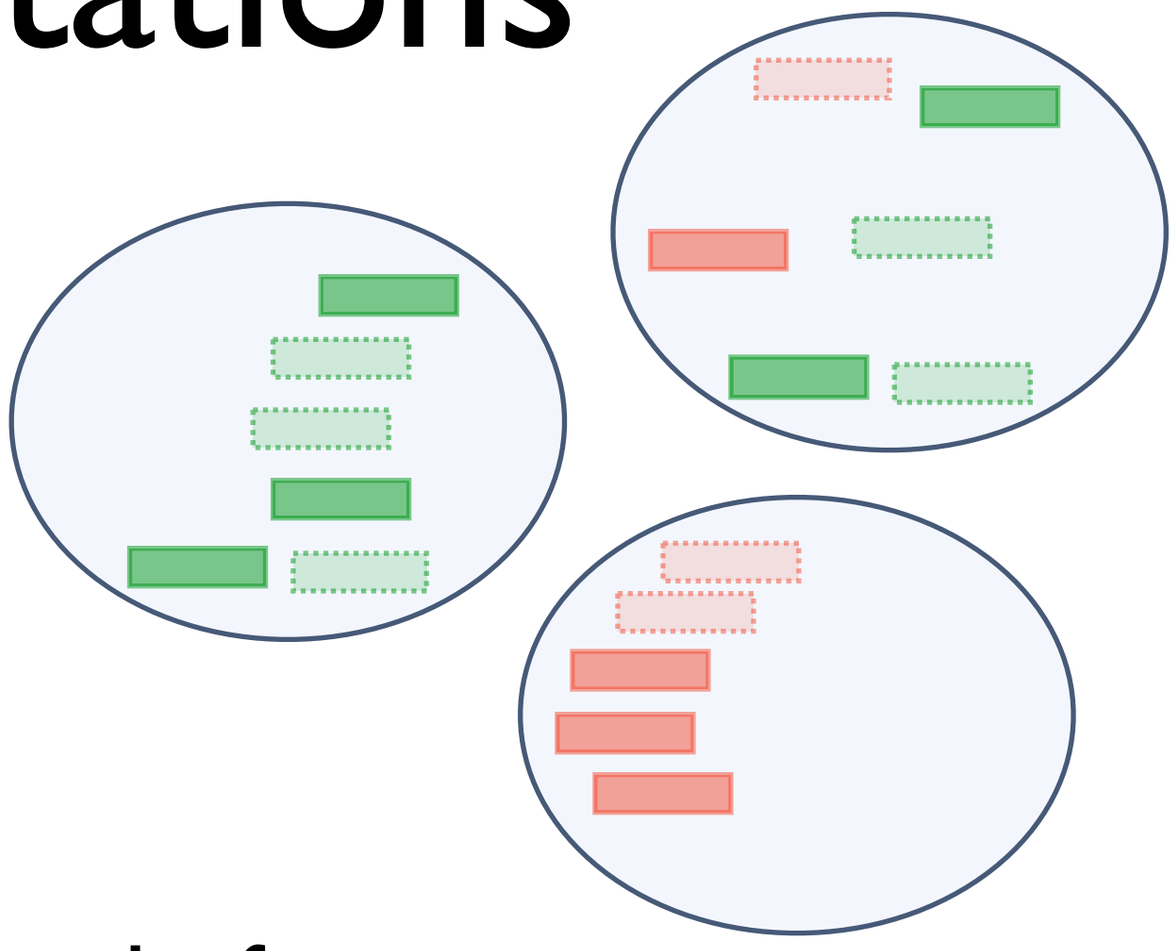






$$p(\textbf{fact}) = \frac{\text{count}(\textbf{fact} \text{ is true})}{\text{Number of interpretations}}$$

# Learning from partial interpretations



- Not all facts observed
- Soft-EM
- use **expected count** instead of **count**
- **P(Q |E) -- conditional queries !**



[Gutmann et al, ECML 11; Fierens et al, TPLP 14]

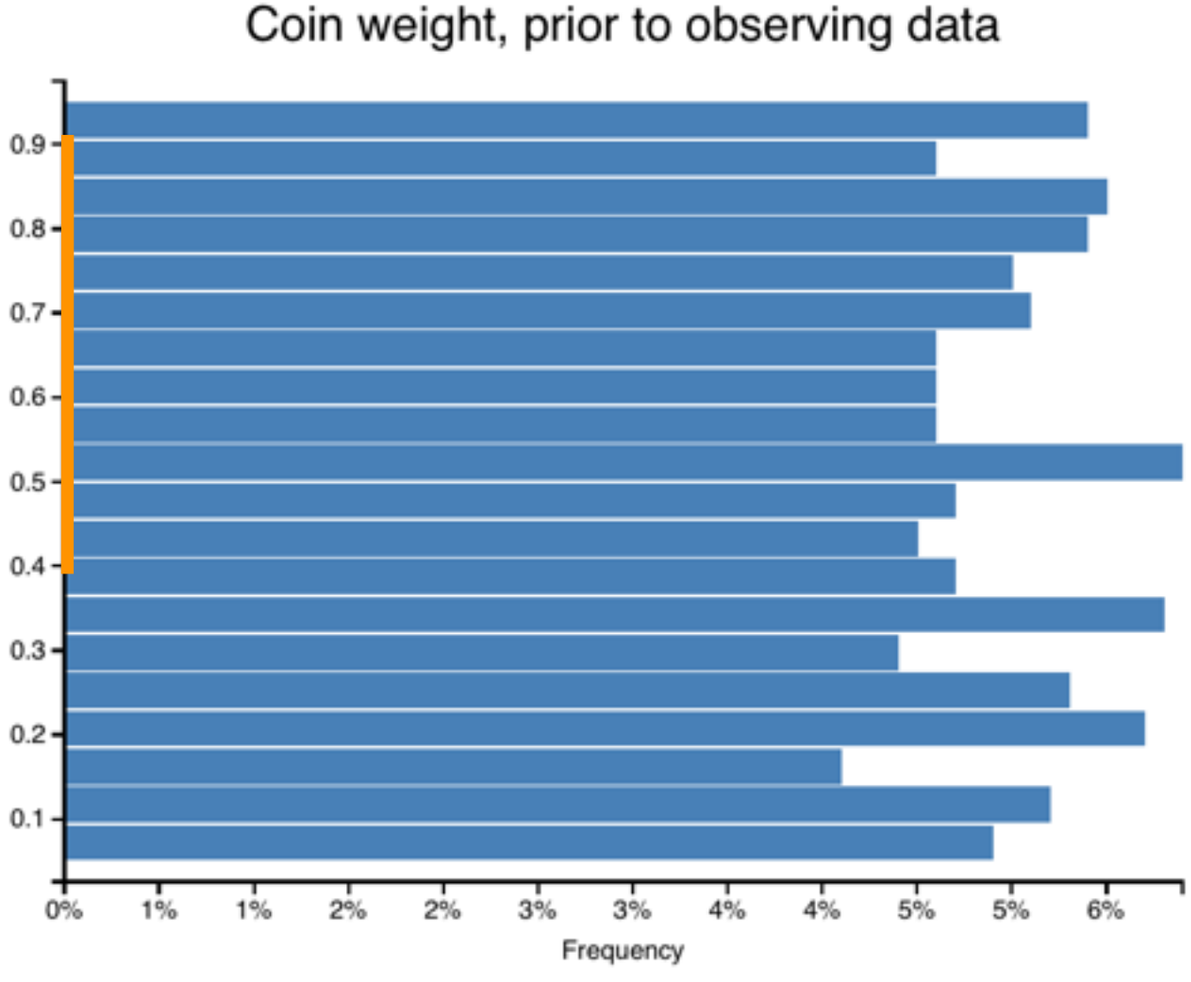
# Bayesian Parameter Learning

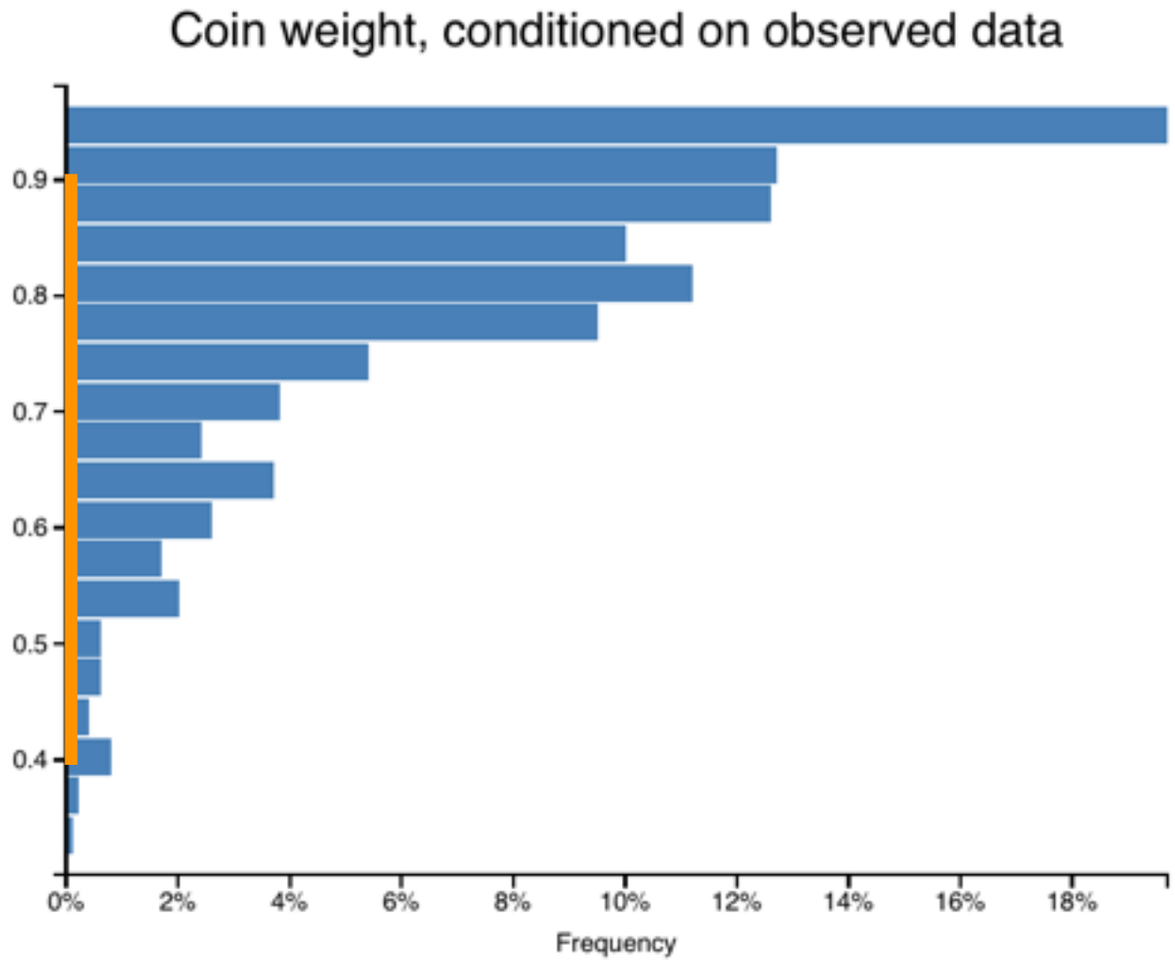
- Learning as inference (e.g., Church)
- Prior distributions for parameters
- Given data, find most likely parameter values

# Example

[from probmods.org]

- Flipping a coin with unknown weight
- Prior: uniform distribution on [0,1]
- Observation: 5x heads in a row
- Sampling from Church model:



# ProbLog Example

**prior**

```
0.05::weight(C,0.1); 0.2::weight(C,0.3); 0.5::weight(C,0.5);
                  0.2::weight(C,0.7); 0.05::weight(C,0.9) :- coin(C).
```

```
Param::toss(_,Param,_).
heads(C,R) :- weight(C,Param),toss(C,Param,R).
tails(C,R) :- weight(C,Param),\+toss(C,Param,R).

data(C,[]).
data(C,[h|R]) :- heads(C,R), data(C,R).
data(C,[t|R]) :- tails(C,R), data(C,R).
```

```
coin(c1).
coin(c2).
param(0.1).
param(0.3).
param(0.5).
param(0.7).
param(0.9).
```

```
query(weight(C,X)) :- coin(C),param(X).
```

**ask for posterior**

```
evidence(data(c1,[h,h,h,h,h,h,h,h,h,h,h,h,h,h]),true).
evidence(data(c2,[h,t,h,h,h,h,h,t,t,h,t,t,h]),true).
```

**data**

# ProbLog Example

```
0.05::weight(C,0.1); 0.2::weight(C,0.3); 0.5::weight(C,0.5);
                    0.2::weight(C,0.7); 0.05::weight(C,0.9) :- coin(C).
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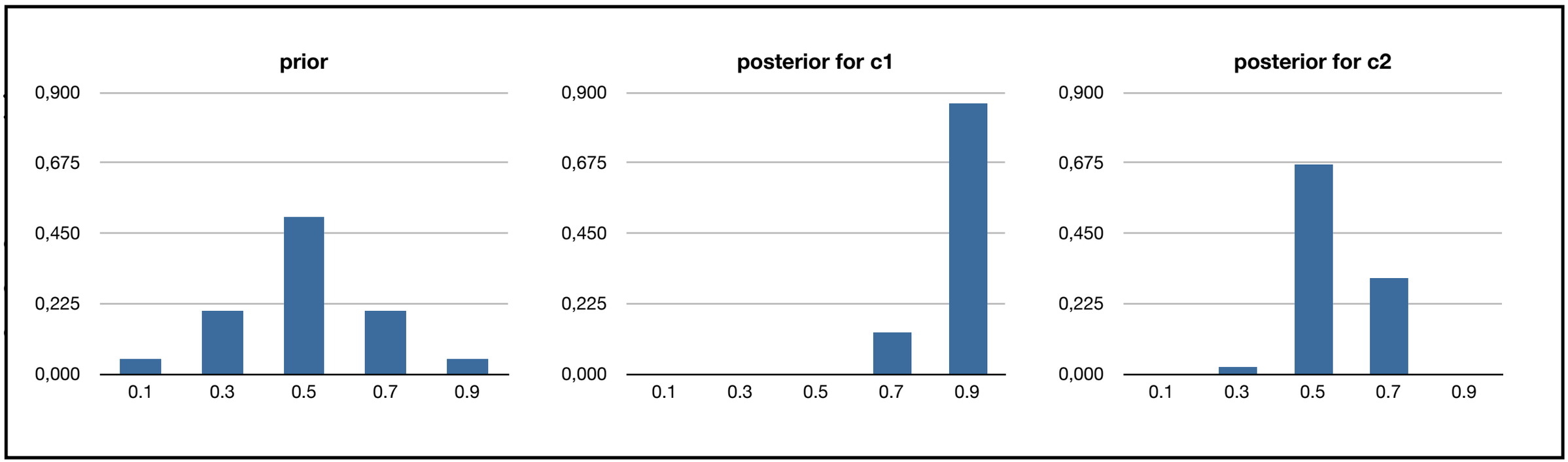


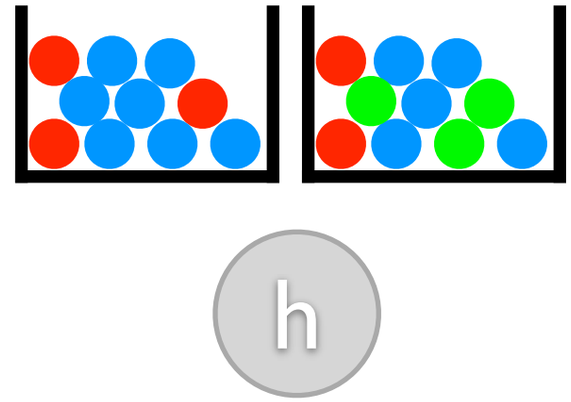

```
query(weight(C,X)) :- coin(C),param(X).

evidence(data(c1,[h,h,h,h,h,h,h,h,h,h,h,h,h,h]),true).
evidence(data(c2,[h,t,h,h,h,h,h,t,t,h,t,t,h]),true).
```

# Some Probabilistic Programming Languages outside LP

- IBAL [Pfeffer 01]
- Figaro [Pfeffer 09]
- Church [Goodman et al 08 ]
- BLOG [Milch et al 05]
- Venture [Mansingha et al.]
- Anglican and Probabilistic-C [Wood et al].
- and many more appearing recently

Church by example:

# A bit of gambling




- toss (biased) coin & draw ball from each urn
- win if (heads and a red ball) or (two balls of same color)

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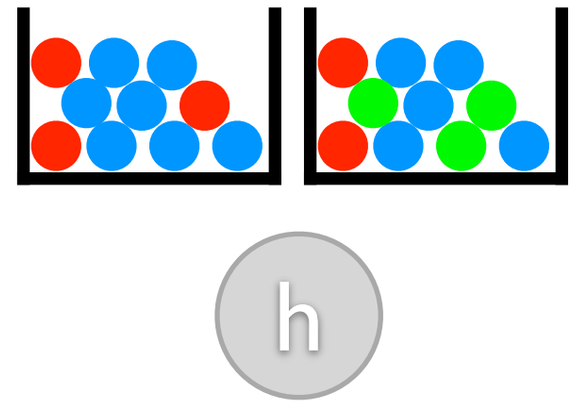


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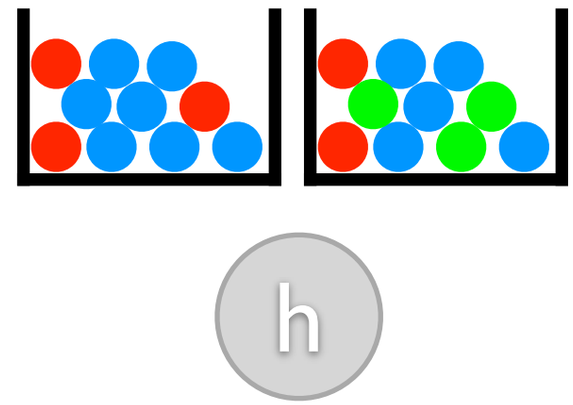


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(define heads (mem (lambda () (flip 0.4))))
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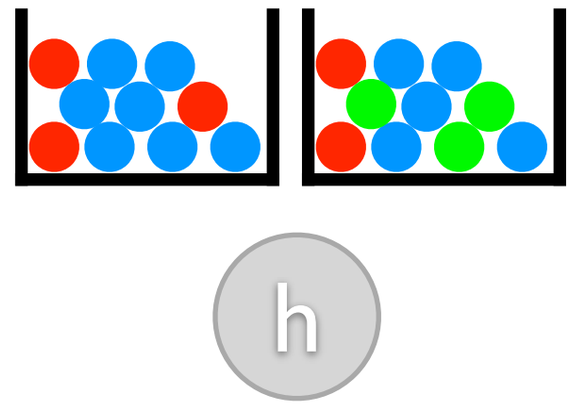


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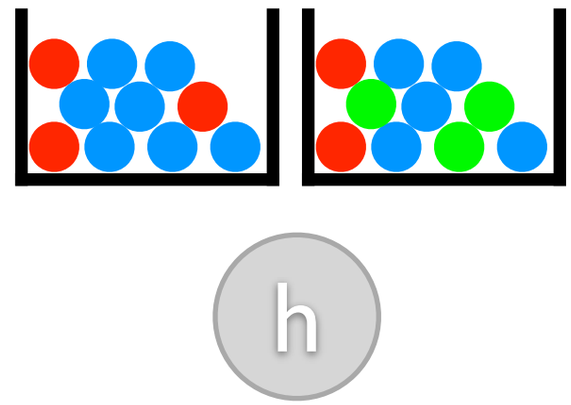


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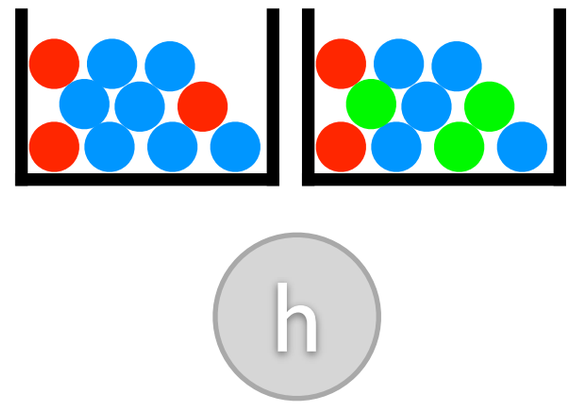


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(define color2 (mem (lambda ()
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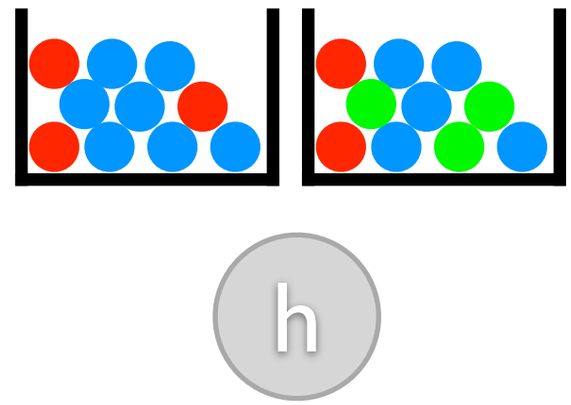
Church by example:

# A bit of gambling

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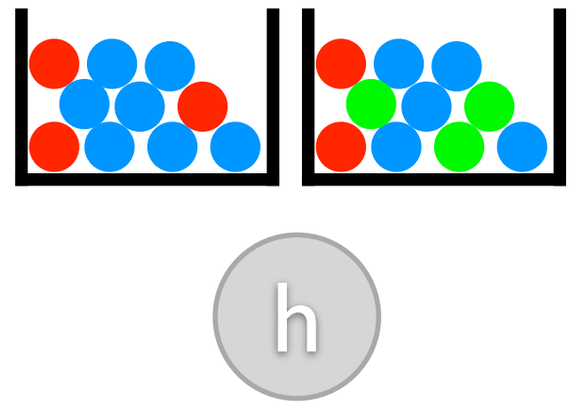


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```

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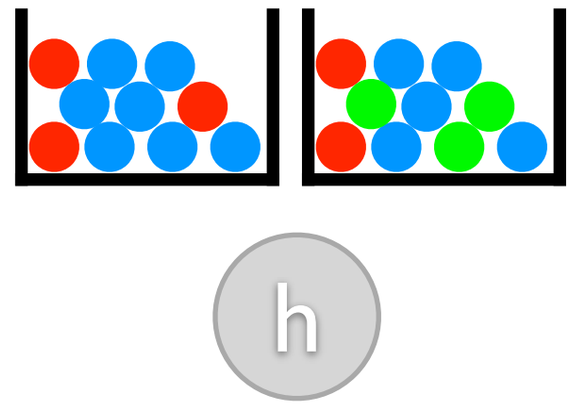


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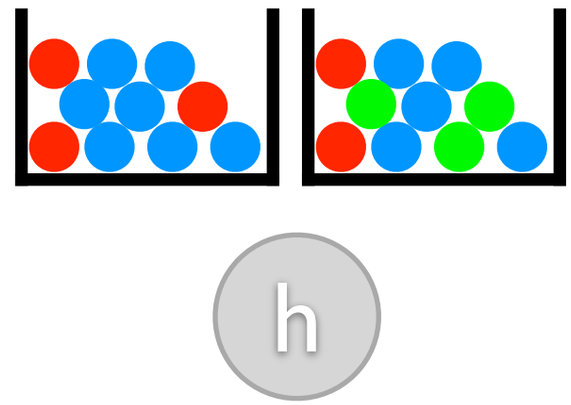


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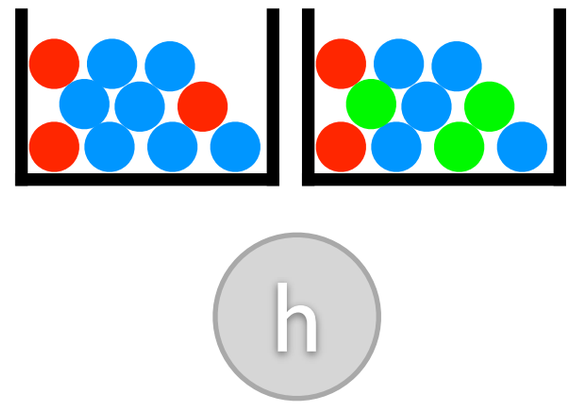


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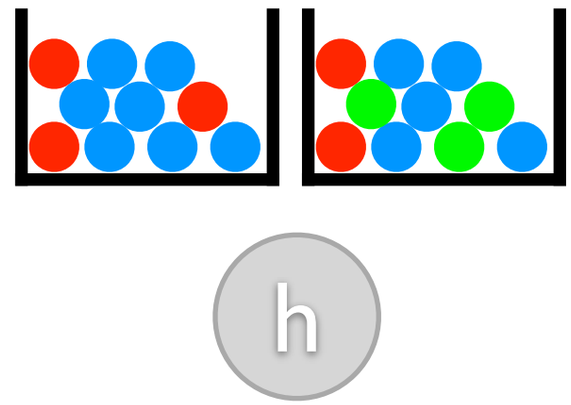


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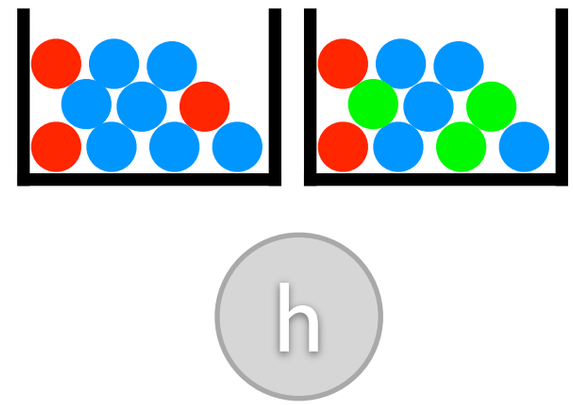


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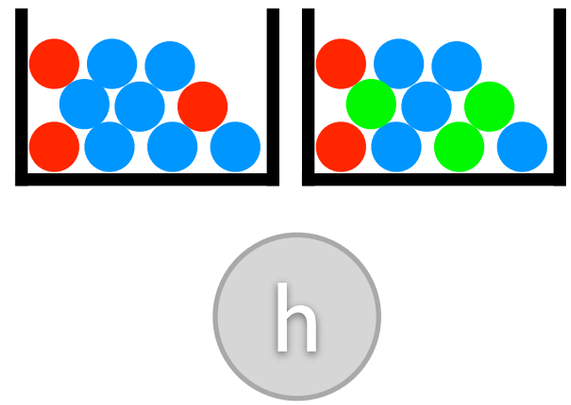


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```
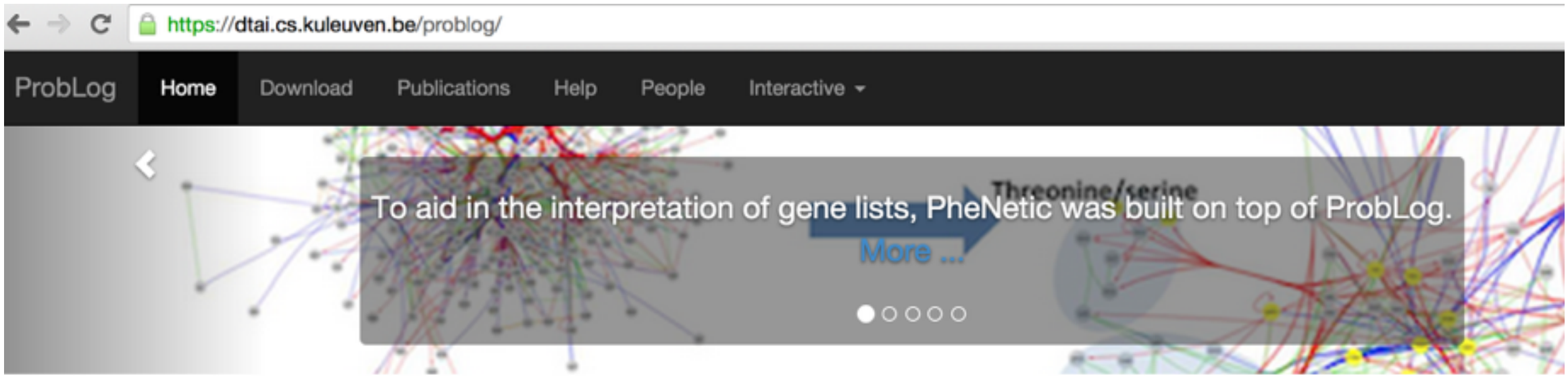

# Probabilistic Programming Summary

- Programming language + random primitives
- probabilistic generative model
- sampling or exact inference
- learning
- increasing number of probabilistic programming languages using various underlying paradigms
- Challenge : inference & scaling up …




# Introduction.

Probabilistic logic programs are logic programs in which some of the facts are annotated with probabilities.

ProbLog is a tool that allows you to intuitively build programs that do not only encode **complex interactions** between a large sets of **heterogenous components** bu **uncertainties** that are present in real-life situations.

The engine tackles several tasks such as computing the marginals given evidence and learning from (partial) interpretations. ProbLog is a suite of efficient algorithms tasks. It is based on a conversion of the program and the queries and evidence to a weighted Boolean formula. This allows us to reduce the inference tasks to well-s weighted model counting, which can be solved using state-of-the-art methods known from the graphical model and knowledge compilation literature.

# The Language. Probabilistic Logic Programming.

ProbLog makes it easy to express complex, probabilistic models.

```
0.3::stress(X) :- person(X).
0.2::influences(X,Y) :- person(X), person(Y).

smokes(X) :- stress(X).
smokes(X) :- friend(X,Y), influences(Y,X), smokes(Y).
```